

MONTEREY SCHOOL
MONTEREY, CALIFORNIA 93945-8001

NAVAL POSTGRADUATE SCHOOL

Monterey, California



THESIS

W 644329

ADAPTIVE ALGORITHMS FOR TWO
DIMENSIONAL FILTERING

by

Steven L. Wilstrup

September 1988

Thesis Advisor:

Murali Tummala

Approved for public release; distribution is unlimited

T242439

classified

Security classification of this page

REPORT DOCUMENTATION PAGE				
Report Security Classification Unclassified			1b Restrictive Markings	
Security Classification Authority			3 Distribution/Availability of Report	
Declassification Downgrading Schedule			Approved for public release; distribution is unlimited.	
Performing Organization Report Number(s)			5 Monitoring Organization Report Number(s)	
Name of Performing Organization Naval Postgraduate School		6b Office Symbol (if applicable) 32	7a Name of Monitoring Organization Naval Postgraduate School	
Address (city, state, and ZIP code) Monterey, CA 93943-5000			7b Address (city, state, and ZIP code) Monterey, CA 93943-5000	
Name of Funding/Sponsoring Organization		8b Office Symbol (if applicable)	9 Procurement Instrument Identification Number	
Address (city, state, and ZIP code)			10 Source of Funding Numbers	
			Program Element No	Project No
			Task No	Work Unit Accession No
Title (Include security classification) ADAPTIVE ALGORITHMS FOR TWO DIMENSIONAL FILTERING				
Personal Author(s) Steven L. Wilstrup				
Type of Report Master's Thesis		13b Time Covered From To	14 Date of Report (year, month, day) September 1988	15 Page Count 75
Supplementary Notation The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.				
Cosati Codes		18 Subject Terms (continue on reverse if necessary and identify by block number)		
Id	Group	Subgroup	Adaptive, Filter, Wiener, LMS, RLS, Noise canceler, ALE	
Abstract (continue on reverse if necessary and identify by block number)				
In this thesis, an adaptive two dimensional least mean squares (LMS) algorithm and a recursive least squares (RLS) algorithm are developed from the one dimensional algorithms. Design of the two dimensional LMS and RLS algorithms are based on the results of a two dimensional system identification model which was used in testing the algorithms. Application of the two algorithms is demonstrated through computer simulation in which the adaptive filters are employed in a noise canceler and an adaptive line enhancer and applied to an image processing problem.				
Distribution/Availability of Abstract Unclassified/unlimited <input type="checkbox"/> same as report <input type="checkbox"/> DTIC users			21 Abstract Security Classification Unclassified	
Name of Responsible Individual Murli Tummala			22b Telephone (include Area code) (408) 646-2645	22c Office Symbol 62Tu

FORM 1473,84 MAR

83 APR edition may be used until exhausted
All other editions are obsolete

Security classification of this page

Unclassified

Approved for public release; distribution is unlimited.

Adaptive Algorithms for Two
Dimensional Filtering

by

Steven L. Wilstrup
Lieutenant Commander, United States Navy
M.B.A., University of West Florida, 1983

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
September 1988

ABSTRACT

In this thesis, an adaptive two dimensional least mean squares (LMS) algorithm and a recursive least squares (RLS) algorithm are developed from the one dimensional algorithms. Design of the two dimensional LMS and RLS algorithms are studied for accuracy based on the results of a two dimensional system identification model which was used in testing the algorithms. Application of the two algorithms is demonstrated through computer simulation in which the adaptive filters are employed in a noise canceler and an adaptive line enhancer and applied to an image processing problem.

TABLE OF CONTENTS

I. INTRODUCTION	1
A. OBJECTIVES OF THE THESIS	2
B. ORGANIZATION OF THE THESIS	2
II. TWO DIMENSIONAL ADAPTIVE LEAST MEAN SQUARE ALGORITHM	4
A. ONE DIMENSIONAL WIENER FILTER	4
B. TWO DIMENSIONAL WIENER FILTER	8
C. TWO DIMENSIONAL LEAST MEAN SQUARE	12
D. IMPLEMENTATION	14
1. System Identification	14
2. Adaptive Noise Canceler	15
3. Adaptive Line Enhancer	18
III. TWO DIMENSIONAL ADAPTIVE RECURSIVE LEAST SQUARES AL-	
GORITHM	21
A. ONE DIMENSIONAL RECURSIVE LEAST SQUARES	21
B. TWO DIMENSIONAL RECURSIVE LEAST SQUARES	24
C. IMPLEMENTATION	29
1. System Identification	29
2. Adaptive Noise Canceler/Adaptive Line Enhancer	30
IV. RESULTS AND CONCLUSIONS	33
A. NOISE CANCELER RESULTS	34
B. ADAPTIVE LINE ENHANCER RESULTS	39
C. CONCLUSIONS	40
APPENDIX A. LMS SYSTEM IDENTIFICATION PROGRAM	42
APPENDIX B. LMS ALGORITHM IMPLEMENTED IN A NOISE	
CANCELER	44

APPENDIX C. LMS ALGORITHM IMPLEMENTED IN AN ADAPTIVE
LINE ENHANCER 48

APPENDIX D. RLS SYSTEM IDENTIFICATION 52

APPENDIX E. RLS ALGORITHM IMPLEMENTED IN A NOISE
CANCELER 55

APPENDIX F. RLS ALGORITHM IMPLEMENTED IN AN ADAPTIVE LINE
ENHANCER 59

LIST OF REFERENCES 64

INITIAL DISTRIBUTION LIST 66

LIST OF TABLES

Table 1. NOISE CANCELER RESULTS	38
Table 2. ADAPTIVE LINE ENHANCER RESULTS	40

LIST OF FIGURES

Figure 1.	Tapped Delay Line Filter	5
Figure 2.	Two Dimensional Wiener Filter	9
Figure 3.	Two Dimensional Least Mean Square	13
Figure 4.	Two Dimensional System Identification	15
Figure 5.	LMS System Identification-Rate of Convergence	16
Figure 6.	LMS System Identification-Rate of Convergence	17
Figure 7.	Adaptive Noise Canceler	18
Figure 8.	Adaptive Line Enhancer	19
Figure 9.	One Dimensional RLS Reference (TDL)	22
Figure 10.	Two Dimensional RLS Reference	25
Figure 11.	Two Dimensional RLS System Identification	29
Figure 12.	RLS System Identification Rate of Convergence	31
Figure 13.	RLS System Identification Rate of Convergence	32
Figure 14.	Original Image	33
Figure 15.	Image plus Additive Noise	34
Figure 16.	Restored Image: Noise Canceler/LMS (One pass)	35
Figure 17.	Restored Image: Noise Canceler/LMS (Two passes)	36
Figure 18.	Restored Image: Noise Canceler/RLS (One pass)	36
Figure 19.	Restored Image: Noise Canceler/LMS (different μ)	37
Figure 20.	Restored Image: Noise Canceler/LMS (normalized μ)	37
Figure 21.	Restored Image: Noise Canceler/LMS (3 by 3)	38
Figure 22.	Restored Image: Adaptive Line Enhancer/LMS (One pass)	39
Figure 23.	Restored Image: Adaptive Line Enhancer/RLS (One pass)	40

I. INTRODUCTION

The area of digital signal processing has experienced a rapid growth in the last decade. Reasons for this have been the tremendous advances in integrated circuit technology, and some significant developments in digital processing techniques achieved during this period. Included in these developments are methods which extend certain one dimensional digital signal processing techniques to two dimensions. This extension is by no means trivial. Three significant factors must be considered:

1. more degrees of freedom are available in a two dimensional system; this gives a system designer more flexibility than the one dimensional case,
2. one dimensional problems generally involve considerably less data than two dimensional problems, and
3. the mathematical methods for handling two dimensional systems are generally less complete than those for one dimension.

As the techniques for processing multidimensional data have improved, the applications of digital signal processing have spread from one dimensional to two dimensional, to n -dimensional data. There exists many physical phenomena that inherently depend on two or more independent variables. In the prediction of weather and in seismic analysis, the data depends on more than one independent variable. Moreover, data originating from one dimensional processes can, in some cases, be considered two dimensional. For instance, data from periodic or cyclic processes can be represented as two dimensional arrays by using their periodicity.

Besides the general applicability of two dimensional signal processing in the above cases, other areas which have experienced significant growth in recent years include radar, sonar, and radio astronomy. The two dimensional processing of images is also a very important one. Images depend on two spatial variables, and are continuous in nature. However, if we digitize them and assume linear models in their formation, distortion, and recording we then have techniques that can be used in their processing. Depending on the applications, different processing techniques are used, notably: enhancement and restoration of images, and segmentation and encoding of images. For details, see References 1,2,3, 4 .

This brief discussion about applications of two dimensional digital signal processing shows that they are found in a wide variety of fields. In this thesis, we are interested in

extending one dimensional adaptive filtering techniques to two dimensions and then applying this in the area of image restoration. Once the transition from one to two dimensions is understood, the extension to n-dimensional signal processing is fairly straightforward.

A. OBJECTIVES OF THE THESIS

The use of the Wiener filter has proven to be a very powerful tool in the area of image restoration [Refs. 1,2, 4]. However, one disadvantage is the fact that the Wiener filter operates under the assumption that the image is stationary which generally is not the case in an image processing problem. In one dimensional signal processing the Wiener filter concept has provided a basis for various adaptive filtering algorithms. Within these adaptive algorithms, the filter possesses characteristics which can be modified to achieve some end or objective and is usually assumed to accomplish this "adaptation" automatically without the need for substantial intervention by the user. The adaptive filter can "learn" the signal characteristics when first turned on and thereafter can track changes in these characteristics. The first objective of this thesis is to examine a two dimensional Wiener model for image restoration which can then be extended to adaptive algorithms. Due to its relatively low computational requirements and the fact that it will work in a variety of signal environments, the least mean square (LMS) algorithm will be investigated first.

The second objective of this thesis is to develop a second two dimensional adaptive algorithm. In this case, we will work with the recursive least square (RLS) algorithm which offers faster convergence than the gradient-search-type algorithms but generally involves a greater cost per data sample and more numerical difficulties.

The final objective is to implement the LMS and RLS algorithms within a system identification model, a noise canceler, and an adaptive line enhancer. Each algorithm possesses several variants and various parameters which can be modified. A representative sample of the various outputs will be examined and the results compared in order to see which provides the optimum solution under given conditions.

B. ORGANIZATION OF THE THESIS

Chapter II is designed to review the development of the one dimensional Wiener filter and then extend it to two dimensions where it can be incorporated into a two dimensional LMS adaptive algorithm. Computer simulation of the algorithm within a system identification model is performed and the results are shown. The two dimensional RLS algorithm is derived in Chapter III and again the results of the algorithm

within a computer simulated system identification model are shown. Chapter IV contains the results of implementing the LMS and RLS algorithms in a noise canceler and an adaptive line enhancer. Conclusions concerning the results are also presented.

II. TWO DIMENSIONAL ADAPTIVE LEAST MEAN SQUARE ALGORITHM

In this chapter, we will review the derivation of the one dimensional Wiener filter and then develop an algorithm to extend it to the two dimensional case. We will use the two dimensional Wiener filter to achieve a two dimensional least mean square (LMS) adaptive filter algorithm which will be demonstrated to be useful in image processing by applying the algorithm in a noise canceler mode and in an adaptive line enhancer configuration for the restoration of images corrupted by noise.

A. ONE DIMENSIONAL WIENER FILTER

The problem of estimating one signal from another is one of the most important in signal processing. In many applications, the desired signal is not available or observable directly. Instead, the observable signal is a degraded or distorted version of the original signal. The signal estimation problem is to recover, in the best way possible, the desired signal from its degraded replica. One typical example which we will deal with in this paper is an image recorded by an imaging system that has been corrupted by noise. The problem is to undo the noise induced distortion and restore the original image.

This represents the classic one dimensional problem in communication theory where we must obtain an estimate of a signal of interest, which can be observed in the presence of some additive noise. In other words, the available information about the signal, $s(n)$, is contained in the received signal:

$$u(n) = s(n) + w(n) \quad (2.1)$$

where $w(n)$ is the noise. Therefore, we must process this available signal $u(n)$ through an optimal processor that produces the best possible estimate of $s(n)$.

In order to establish a two dimensional Wiener filter, we must first develop a one dimensional algorithm. This task has been approached from many different directions [Refs. 5,6,7]. The formulation by Haykin [Ref. 8] provides the most logical extension to two dimensions. First, we consider a tapped delay line filter similar to Figure 1 on page 5. The filter consists of a set of delay elements, a corresponding set of adjustable tap gains or coefficients $h(1), h(2), \dots, h(M)$ connected to the tap inputs, and a set of adders

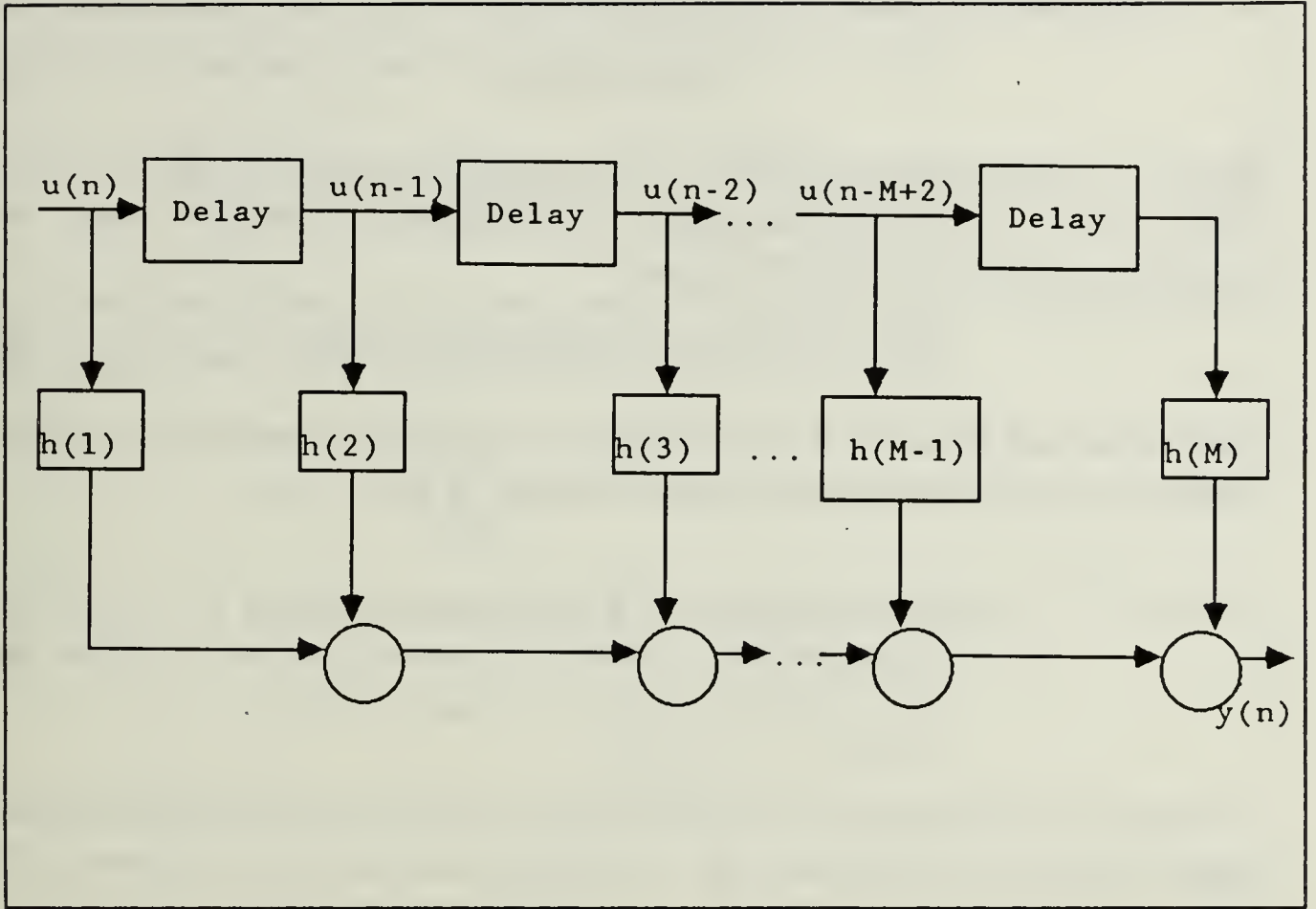


Figure 1. Tapped Delay Line Filter

for summing the resultant outputs. The filter is driven by a random time series producing the sequence $u(n), u(n-1), \dots, u(n-M+1)$ as the M tap inputs of the filter.

We can express the signal produced at the filter output, $y(n)$, by the convolution sum:

$$y(n) = \sum_{k=1}^M h(k)u(n-k+1). \quad (2.2)$$

We desire a filter which in some way minimizes the difference between some desired response, $d(n)$, and the corresponding value of the actual filter output. This difference can be denoted as

$$e(n) = d(n) - y(n) \quad (2.3)$$

where $e(n)$ is called the error signal. In Wiener theory, the filter is optimized by minimizing the mean-square value of this error signal, $e(n)$.

Let the mean-square value of the error be denoted by

$$MSE = E \{e^2(n)\} \quad (2.4)$$

where $E \{.\}$ is the expectation operator. This mean-square value is a real and positive scalar quantity, representing the average normalized power of the error signal, $e(n)$. Substituting Equation (2.3) into (2.4) yields

$$MSE = E \{d^2(n)\} - 2 E \{d(n)y(n)\} + E \{y^2(n)\}. \quad (2.5)$$

Next, substituting Equation (2.2) into Equation (2.5) and then interchanging the orders of summation and expectation in the last two terms, we get

$$\begin{aligned} MSE = & E \{d^2(n)\} - 2 \sum_{k=1}^M h(k) E \{d(n)u(n-k+1)\} \\ & + \sum_{k=1}^M \sum_{m=1}^M h(k)h(m) E \{u(n-k+1)u(n-m+1)\}. \end{aligned} \quad (2.6)$$

Assuming that the input signal $u(n)$ and the desired response $d(n)$ are jointly stationary, the three terms on the right-hand side of the above equation may be interpreted as follows:

1. The expectation $E \{d^2(n)\}$ is equal to the mean square value of the desired response $d(n)$:

$$P_d = E \{d^2(n)\}. \quad (2.7)$$

2. The expectation $E \{d(n)u(n-k+1)\}$ is equal to the cross-correlation function of the desired response $d(n)$ and the input signal $u(n)$ for the lag of $k-1$. We can therefore write the single summation term on the righthand side of Equation (2.6) as follows:

$$\sum_{k=1}^M h(k) E \{d(n)u(n-k+1)\} = \sum_{k=1}^M h(k) p(k-1). \quad (2.8)$$

3. Finally, the expectation $E \{u(n-k+1)u(n-m+1)\}$ is equal to the autocorrelation function of the input signal $u(n)$ for the lag of $m-k$:

$$r(m-k) = E \{u(n-k+1)u(n-m+1)\}. \quad (2.9)$$

Accordingly, we can rewrite the double summation term on the righthand side of Equation (2.6) in the form

$$\begin{aligned} & \sum_{k=1}^M \sum_{m=1}^M h(k)h(m) E \{u(n-k+1)u(n-m+1)\} \\ & = \sum_{k=1}^M \sum_{m=1}^M h(k)h(m)r(m-k). \end{aligned} \quad (2.10)$$

Thus, substituting Equations (2.7), (2.8), and (2.10) into Equation (2.6), we find that the expression for the mean square error may be rewritten in the form

$$MSE = P_d - 2 \sum_{k=1}^M h(k)p(k-1) + \sum_{k=1}^M \sum_{m=1}^M h(k)h(m)r(m-k). \quad (2.11)$$

By differentiating Equation (2.11) with respect to $h(k)$ and setting it equal to zero, we have the following set of M simultaneous equations

$$p(k-1) = \sum_{m=1}^M h_0(m)r(m-k), \quad k = 1, 2, \dots, M. \quad (2.12)$$

This system of M simultaneous equations is called the normal equations with optimum filter coefficients as the unknowns. With the following definitions,

$$\mathbf{h}_0 = \begin{bmatrix} h_0(1) \\ h_0(2) \\ \vdots \\ \vdots \\ h_0(M) \end{bmatrix} \quad (2.13)$$

$$\mathbf{p} = \begin{bmatrix} p(0) \\ p(1) \\ \vdots \\ \vdots \\ p(M-1) \end{bmatrix} \quad (2.14)$$

$$\mathbf{R} = \begin{bmatrix} r(0) & r(1) & \dots & r(M-1) \\ r(1) & r(0) & \dots & r(M-2) \\ \vdots & \vdots & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ r(M-1) & r(M-2) & \dots & r(0) \end{bmatrix} \quad (2.15)$$

we can rewrite the normal equations of Equation (2.12) in matrix form as

$$\mathbf{p} = \mathbf{R} \mathbf{h}_0. \quad (2.16)$$

This represents the discrete-time version of the Wiener-Hopf equation.

B. TWO DIMENSIONAL WIENER FILTER

The following derivation parallels the development by Hadhoud and Thomas, however this research and simulation were completed separately and prior to the publication of Reference 9. In order to be applicable for an image processing problem, we must extend the formulation in the previous section to two dimensions [Refs. 10,11]. This is accomplished by developing a basic two dimensional Wiener filter as shown in Figure 2 on page 9. Within this filter, we use two input images, the reference array X and the primary input array D . The primary input image D is a two dimensional array which represents the ideal image plus additive noise, while the reference image X is noise that is assumed to be correlated to the noise in the primary input. Both the input arrays are $M \times M$ in dimension. The Wiener filter is an $N \times N$ causal FIR filter with a set of weights W_j defined as

$$W_j = \begin{bmatrix} W_j(0,0) & W_j(0,1) & \dots & W_j(0,N-1) \\ W_j(1,0) & W_j(1,1) & \dots & W_j(1,n-1) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ W_j(N-1,0) & W_j(N-1,1) & \dots & W_j(N-1,N-1) \end{bmatrix} \quad (2.17)$$

which minimize the mean of the squared error, e_j , between the filter output and the desired input D . We designate j as the iteration number given by $j = mM + n$ where m and n take on the values from 0 to $M-1$. Just as in the one dimensional case, the filter output, $y(m,n)$, is the convolution sum of the filter mask and the reference input X which is given by

$$y(m,n) = \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} W_j(l,k) X(m-l, n-k). \quad (2.18)$$

During the j th iteration the input from array X is represented by X_j where

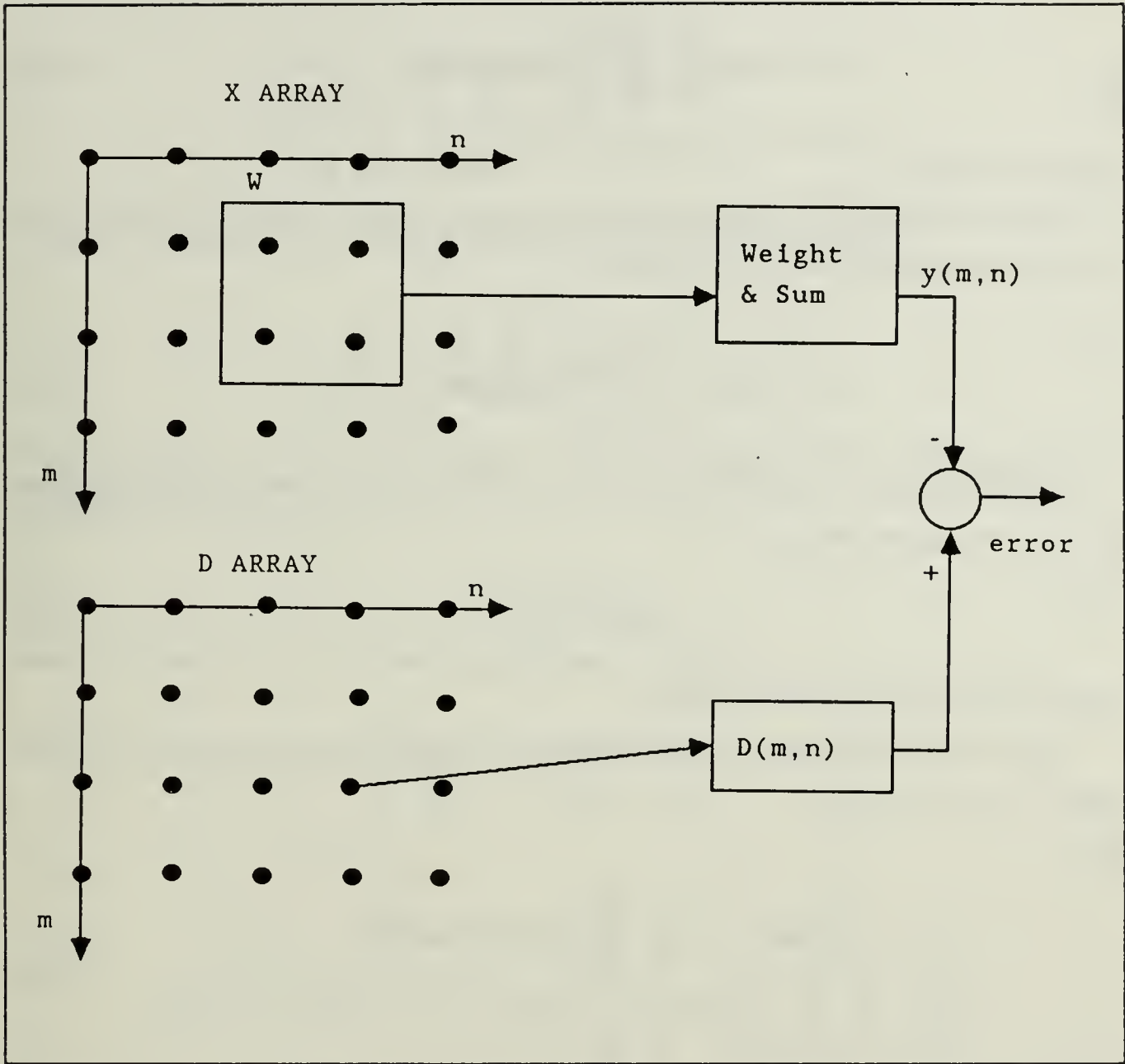


Figure 2. Two Dimensional Wiener Filter

$$X_j = \begin{bmatrix} X(m,n) & X(m,n-1) & \dots & X(m,n-N+1) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ X(m-N+1,n) & X(m-N+1,n-1) & \dots & X(m-N+1,n-N+1) \end{bmatrix} \quad (2.19)$$

Using the Equations (2.17) and (2.19), Equation (2.18) can now be written as

$$y(m,n) = \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} W_j(l,k) X_j(l,k) \quad (2.20)$$

for the j th iteration.

This output $y(m,n)$ can now be subtracted from one pixel $D(m,n)$ of the array D to produce the error signal at the j th iteration

$$e_j = D(m,n) - \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} W_j(l,k) X(m-l, n-k). \quad (2.21)$$

Since the purpose of the Wiener filter is to provide a set of weights which minimize the MSE, we can denote it as

$$MSE = E\{e_j^2\} \quad (2.22)$$

where $E\{.\}$ is the expectation operator.

Using a mathematical derivation similar to the one dimensional case of the previous section, we can see that

$$\begin{aligned} e_j^2 &= D^2(m,n) - 2 \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} W_j(l,k) D(m,n) X(m-l, n-k) \\ &+ \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} W_j(l,k) W_j(p,q) X(m-l, n-k) X(m-p, n-q). \end{aligned} \quad (2.23)$$

Substituting Equation (2.23) into Equation (2.22) yields

$$\begin{aligned} MSE &= E[D^2(m,n)] - 2 \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} W_j(l,k) E[D(m,n) X(m-l, n-k)] \\ &+ \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} W_j(l,k) W_j(p,q) E[X(m-l, n-k) X(m-p, n-q)]. \end{aligned} \quad (2.24)$$

Defining P as the crosscorrelation matrix between the desired response $D(m,n)$ and the reference input, R as the input autocorrelation matrix, and W as the optimum Wiener weight matrix, we can rewrite Equation (2.24) as

$$\begin{aligned}
MSE = E[D^2(m,n)] &- 2 \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} W_j(l,k) P(l,k) \\
&+ \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} W_j(l,k) W_j(p,q) R(p-l, q-k).
\end{aligned} \tag{2.25}$$

Finally, if we minimize the MSE with respect to $W_j(l,k)$ then we have

$$P(l,k) = \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} W_j(p,q) R(p-l, q-k). \tag{2.26}$$

This equation is the two dimensional equivalent to Equation (2.12). The matrix form of Equation (2.26) is given as

$$\mathbf{P} = \mathbf{R} \mathbf{W} \tag{2.27}$$

where the elements of this equation are defined as follows

$$\mathbf{R} = \begin{bmatrix} [R_0] & [R_1] & [R_2] & \dots & \dots \\ [R_1] & [R_0] & [R_1] & \dots & \dots \\ \cdot & \cdot & \cdot & \dots & \dots \\ \cdot & \cdot & \cdot & \dots & \dots \\ [R_{-N+1}] & [R_{-N+2}] & \dots & \dots & [R_0] \end{bmatrix} \tag{2.28}$$

$$\mathbf{W} = \begin{bmatrix} W_0 \\ W_1 \\ \cdot \\ \cdot \\ W_{N-1} \end{bmatrix} \tag{2.29}$$

$$\mathbf{P} = \begin{bmatrix} P_0 \\ P_1 \\ \cdot \\ \cdot \\ P_{N-1} \end{bmatrix} \tag{2.30}$$

Within the \mathbf{R} matrix in equation (2.28) each element is a block Toeplitz matrix represented by the equation

$$R(p-l, q-k) = E[X(m-l, n-k)X(m-p, n-q)] \quad (2.31)$$

where l, k, p, q range from 0 to $N-1$.

C. TWO DIMENSIONAL LEAST MEAN SQUARE

One means of obtaining an approximate solution for the optimum weight matrix, \mathbf{W} , is the use of a two dimensional LMS algorithm which is depicted in Figure 3 on page 13. This differs from Figure 2 on page 9 in that the error e_j is used to update the filter coefficients before shifting the data window X_j across the reference input for the next iteration. As in the one dimensional LMS algorithm, we are updating the coefficient matrix by adding the present matrix to a change proportional to the negative gradient of the error where the one dimensional instantaneous estimates of the gradient vector are based on sample values of the input and the error signal $e(n)u(n)$ [Ref. 8]. For the two dimensional j th iteration, we define the updated matrix as

$$W_{j+1} = W_j - \mu e_j X_j \quad (2.32)$$

where

- W_{j+1} updated weight matrix
- W_j previous weight matrix
- μ scaler multiplier controlling the rate of convergence and filter stability
- $(e_j)(X_j)$ estimate for the 2-D instantaneous gradient

The previous equation can also be written as

$$W_{j+1}(l, k) = W_j(l, k) + 2 \mu (e_j) X(m-l, n-k). \quad (2.33)$$

These two equations give the two dimensional weight updating algorithm for the LMS adaptive filter. The algorithm we have developed here may be implemented without any form of matrix operations, averaging, or differentiation.

The value of μ may be chosen based on the desired tracking ability, steady-state mean square error, and convergence speed. In signal processing, there are several methods for determining a suitable value, however in many of these cases it requires knowledge of the eigenvalues and eigenvectors of the autocorrelation matrix. In image

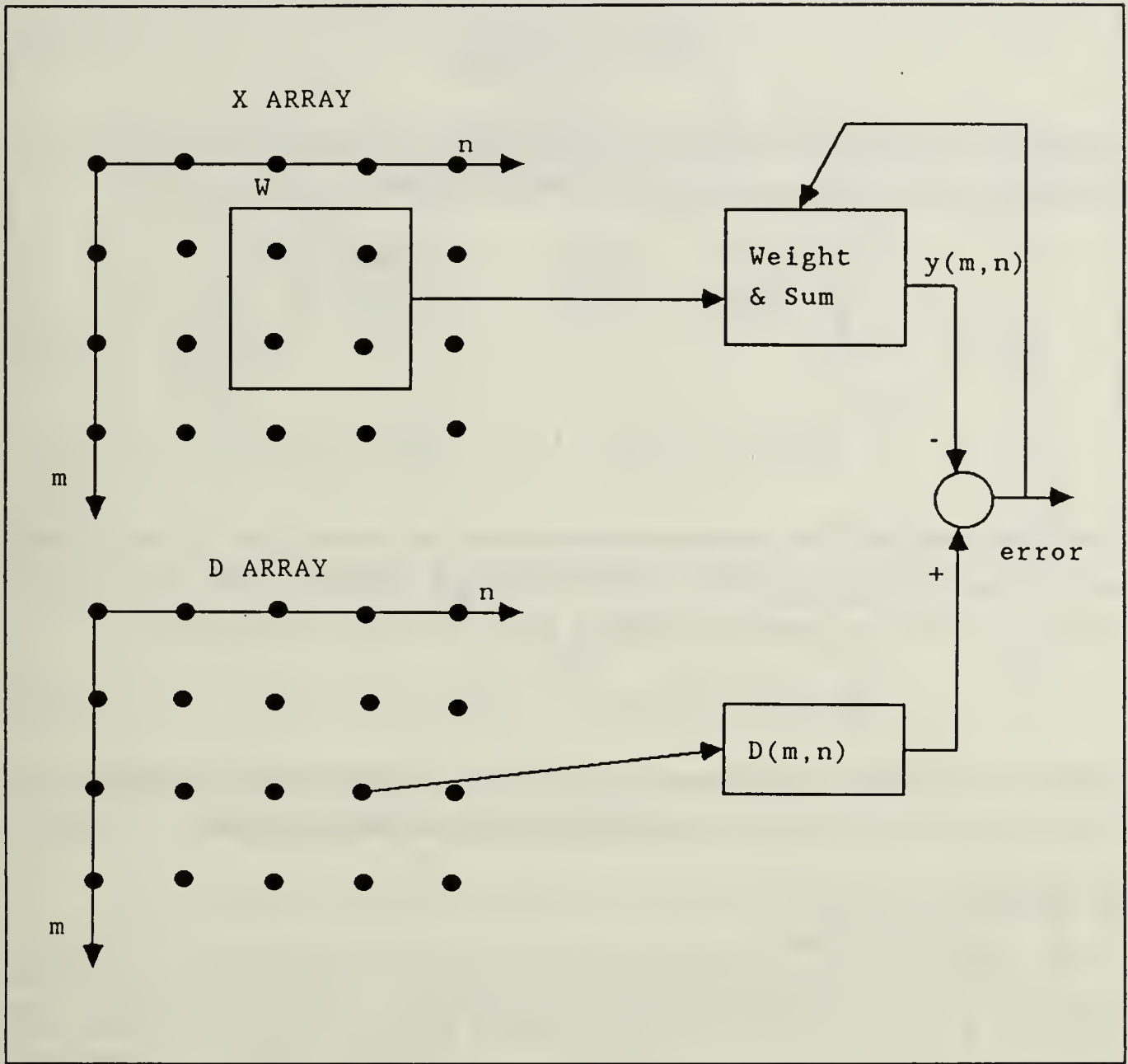


Figure 3. Two Dimensional Least Mean Square

processing, one method which does not require these values for the computation of μ is trial and error based on output image.

An alternate method used in one dimensional design which again does not require a priori knowledge of the autocorrelation matrix, discussed by Bitmead and Anderson [Ref. 12], is the normalized LMS. Using our previous notation, this method can be extended to two dimensions by first considering the two dimensional LMS update equation (2.32). In this equation, we redefine the step-size parameter, μ , for a given l and k as

$$\mu(l,k) = \frac{\alpha}{\beta + \sigma_j^2(l,k)} \quad (2.34)$$

in which α , represents the normalized step size chosen between zero and two, β is another small positive constant, and $\sigma_j^2(l,k)$ is one of the values from the matrix

$$\sigma_j^2 = \begin{bmatrix} \sigma_j^2(0,0) & \sigma_j^2(0,1) & \dots & \sigma_j^2(0,N-1) \\ \sigma_j^2(1,0) & \sigma_j^2(1,1) & \dots & \sigma_j^2(1,N-1) \\ \vdots & \vdots & \dots & \vdots \\ \sigma_j^2(N-1,0) & \sigma_j^2(N-1,1) & \dots & \sigma_j^2(N-1,N-1) \end{bmatrix} \quad (2.35)$$

The element required from the matrix depends upon the current values of l and k being used for equation (2.33). The matrix may be initialized with the values of $(X(m-l, n-k))^2$ and to update the values in σ_j^2 , we use the following equation

$$\sigma_{j+1}^2(l,k) = \rho(\sigma_j^2(l,k)) + (1 - \rho)(X(m-l, n-k))^2 \quad (2.36)$$

where ρ is a weighting factor between zero and one. This ensures that the value of μ does not become large enough to cause the algorithm to become unstable.

D. IMPLEMENTATION

1. System Identification

In order to initially test the two dimensional least mean square algorithm, we established a system identification model shown in Figure 4 on page 15. Within this model the output of the known FIR filter, $d(m,n)$, is defined as

$$\begin{aligned} d(m,n) = & .4 W(m,n) + .6 W(m-1,n) \\ & - .3 W(m,n-1) + .2 W(m-1,n-1). \end{aligned} \quad (2.37)$$

The output of the two dimensional least mean square filter, $y(m,n)$, consists of a set of adjustable coefficients and is defined as

$$\begin{aligned} y(m,n) = & A0 W(m,n) + A1 W(m-1,n) \\ & + A2 W(m,n-1) + A3 W(m-1,n-1). \end{aligned} \quad (2.38)$$

The adaptive filter output $y(m,n)$ is then compared with the known system output $d(m,n)$ to produce an error signal $e(m,n)$, defined as the difference between them.

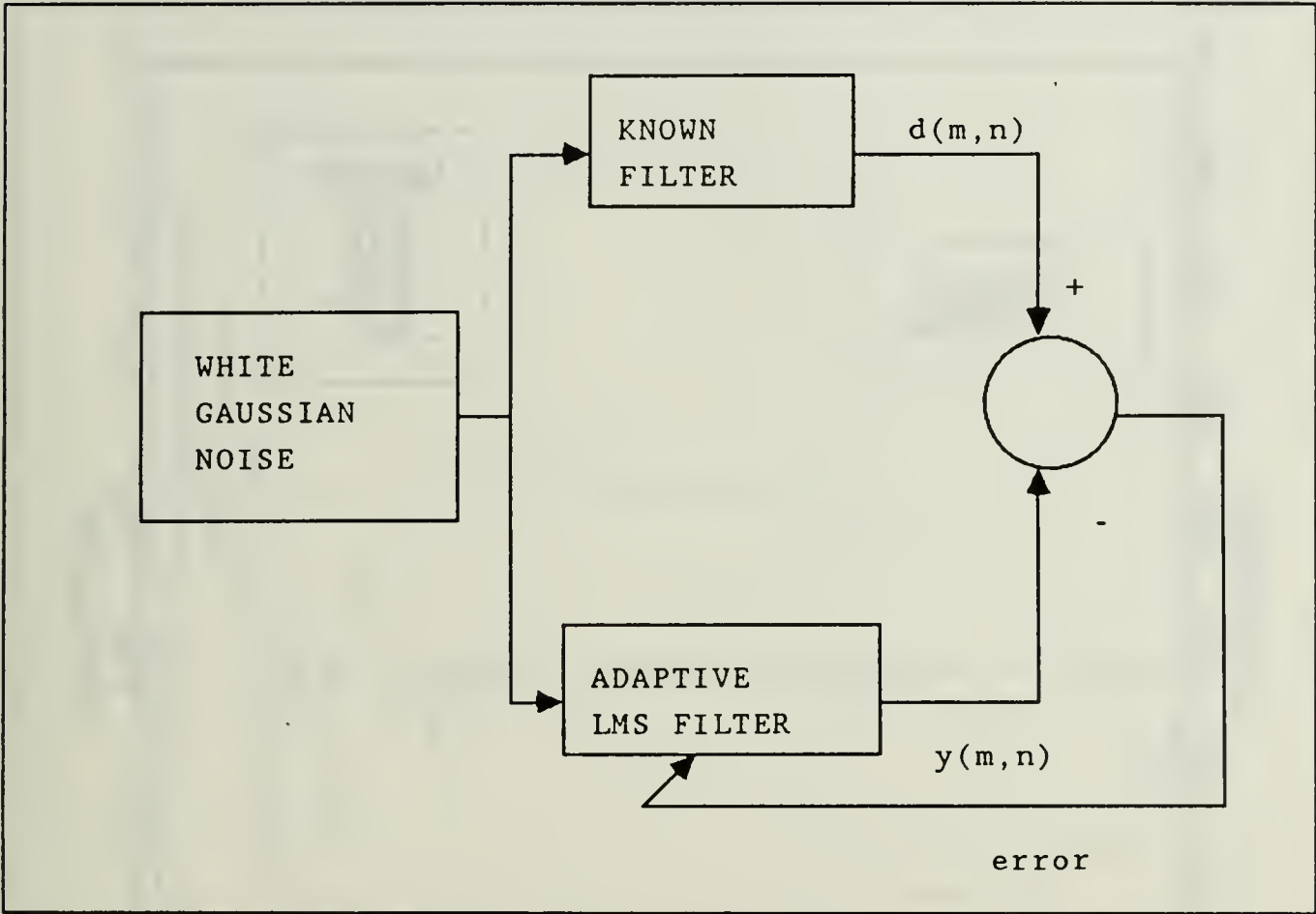


Figure 4. Two Dimensional System Identification

The operation of the adaptive filter is to minimize the error signal $e(m,n)$ by providing an adaptive process for adjusting the coefficients. For this adaptive process we use the update equation (2.32) for the two dimensional least mean square algorithm developed in the previous section

$$W_{j+1} = W_j - \mu e_j X_j \quad (2.32)$$

For this model a 32x32 white gaussian noise matrix was used as the input and the rate of convergence can be seen in Figure 5 on page 16 and Figure 6 on page 17. Following 600 iterations all the coefficients had converged to within 10^{-3} of the actual coefficient value and the error was 3×10^{-5} . A computer program for this system identification model is given in Appendix A.

2. Adaptive Noise Canceler

The usual method of estimating a signal corrupted by additive noise is to pass the composite signal through a filter that tends to suppress the noise while leaving the

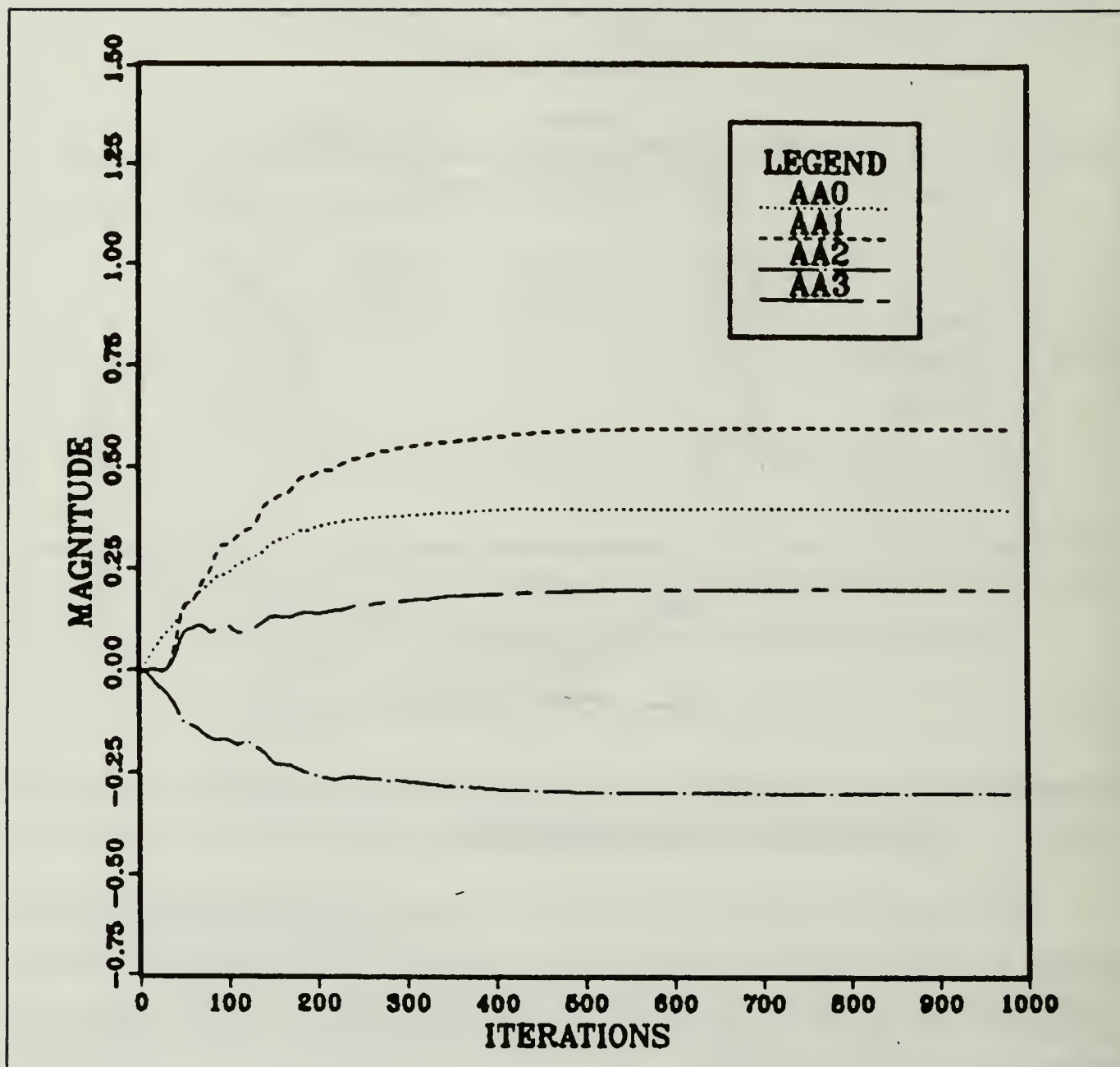


Figure 5. LMS System Identification-Rate of Convergence

signal relatively unchanged. The noise canceler and adaptive line enhancer developed by Widrow [Ref. 13] are well-documented ways of doing that. Adaptive noise canceling is a variation of optimal filtering that is highly advantageous in many applications. It uses an auxiliary or reference input derived from one or more sensors located at points in the noise field where the signal is weak or undetectable. This input is filtered and subtracted from a primary input containing both signal and noise. The reference input and the noise in the primary input are therefore correlated, and as a result the primary noise is attenuated or eliminated by cancellation.

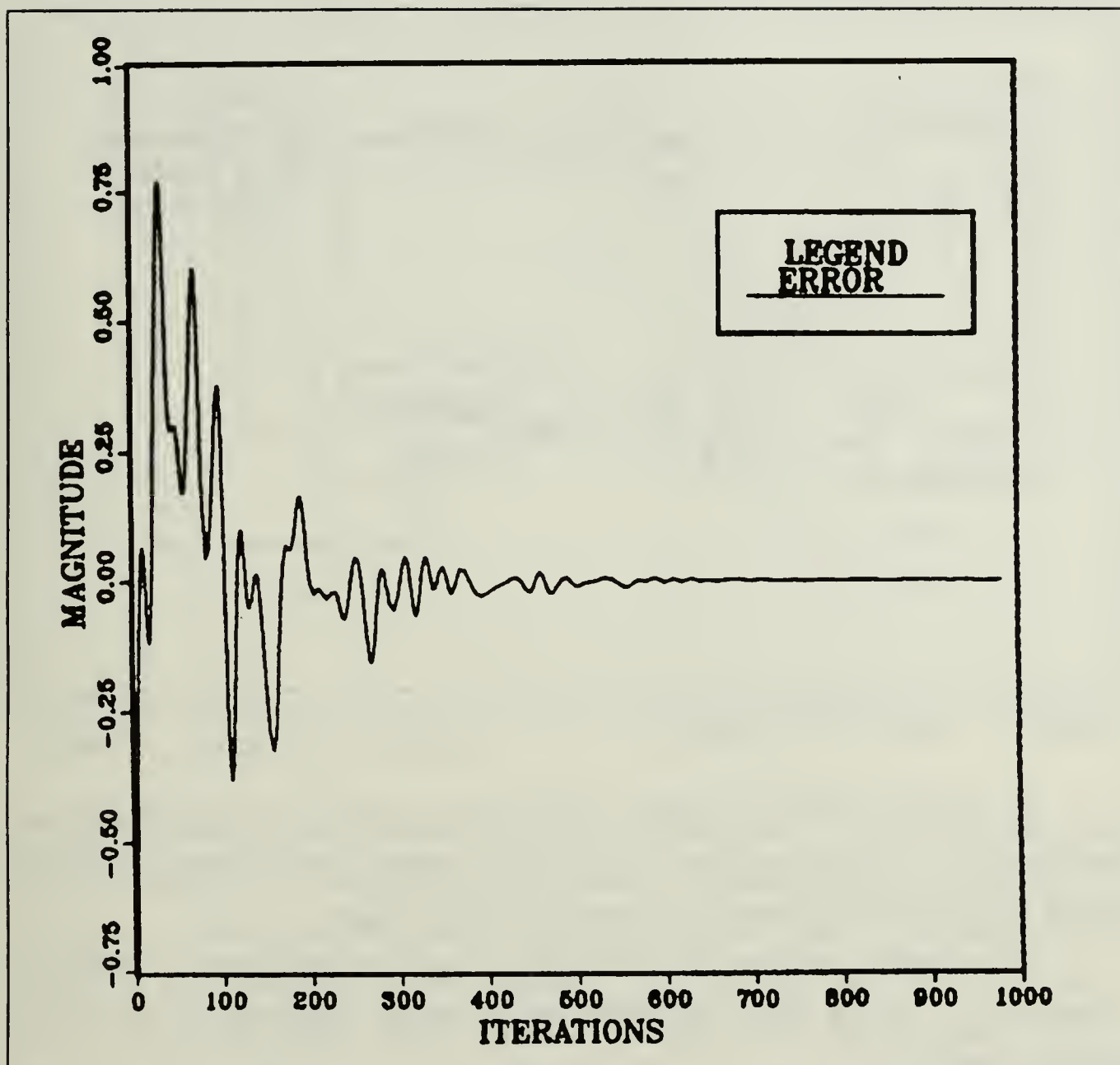


Figure 6. LMS System Identification-Rate of Convergence

The basic noise canceling concept is illustrated in Figure 7 on page 18. A signal is transmitted over a channel to a sensor that receives the signal plus noise, n_0 . The combined signal and noise $s + n_0$ forms the "primary input" to the canceler. A second sensor receives a noise n_1 which is correlated in some way with the noise n_0 . This sensor output provides the "reference input" to the canceler. The noise n_1 is filtered to produce an output y that is a close replica of n_0 . This output is subtracted from the primary input $s + n_0$ to produce the system output $s + n_0 - y$.

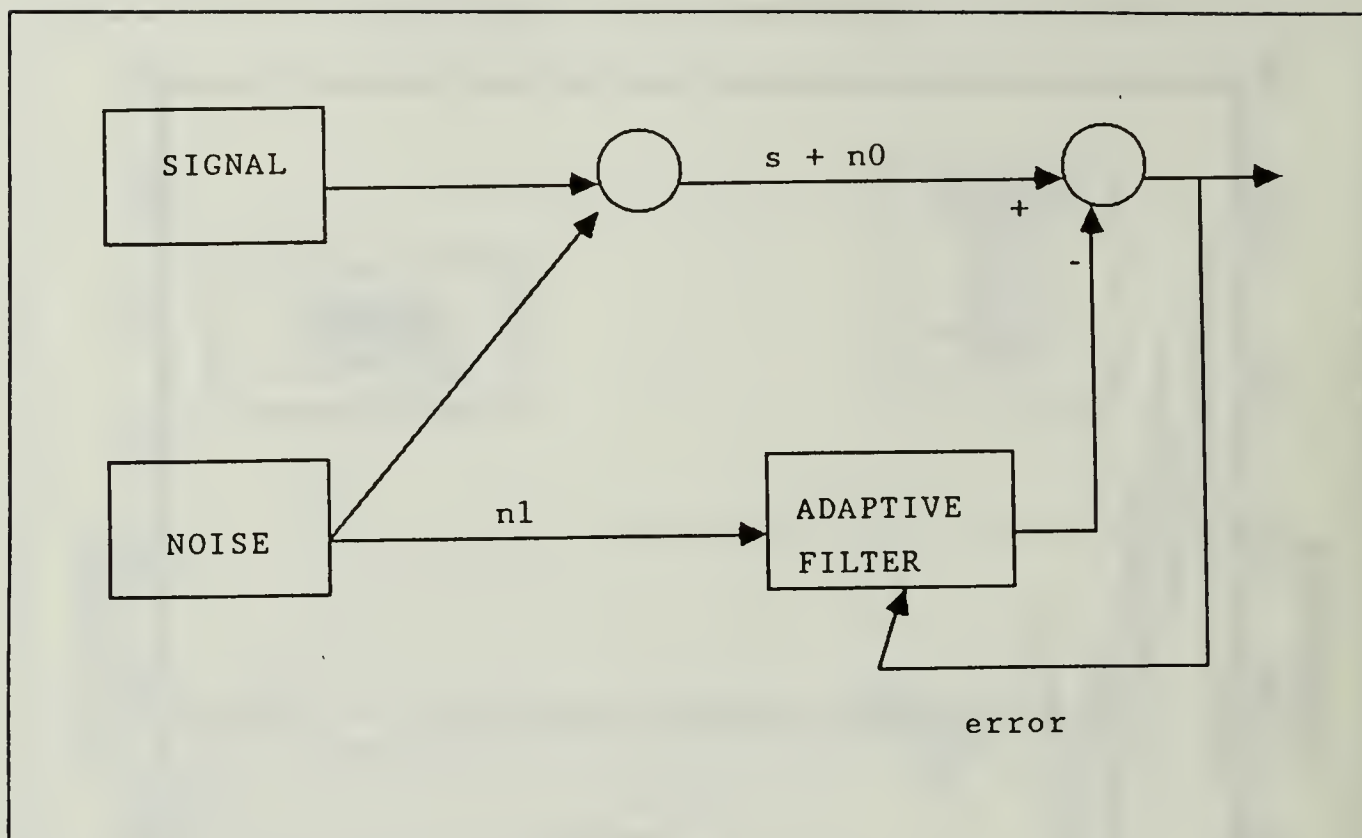


Figure 7. Adaptive Noise Canceler

In the system shown in Figure 7, the reference input is processed by our two dimensional least mean square filter. Thus the filter operates under changing conditions and will readjust itself continuously to minimize the error signal.

The computer program which simulates this noise canceling model is provided in Appendix B. We will discuss the results and conclusions concerning various simulations in Chapter 4.

3. Adaptive Line Enhancer

A special case of adaptive noise canceling is when there is only one signal $x(n)$ available which is contaminated by noise. In such a case, the signal $x(n)$ provides its own reference signal $y(n)$, which is taken to be a delayed replica of $x(n)$: $y(n) = x(n - \Delta)$, as shown in Figure 8 on page 19. The adaptive filter will respond by canceling any components of the main signal $x(n)$ that are in any way correlated with the secondary signal $y(n) = x(n - \Delta)$. Suppose the signal $x(n)$ consists of two components: a narrowband component that has long-range correlations and a broadband component which will tend to have short-range correlations. One of these could represent the desired signal and the other an undesired interfering noise. Suppose that the

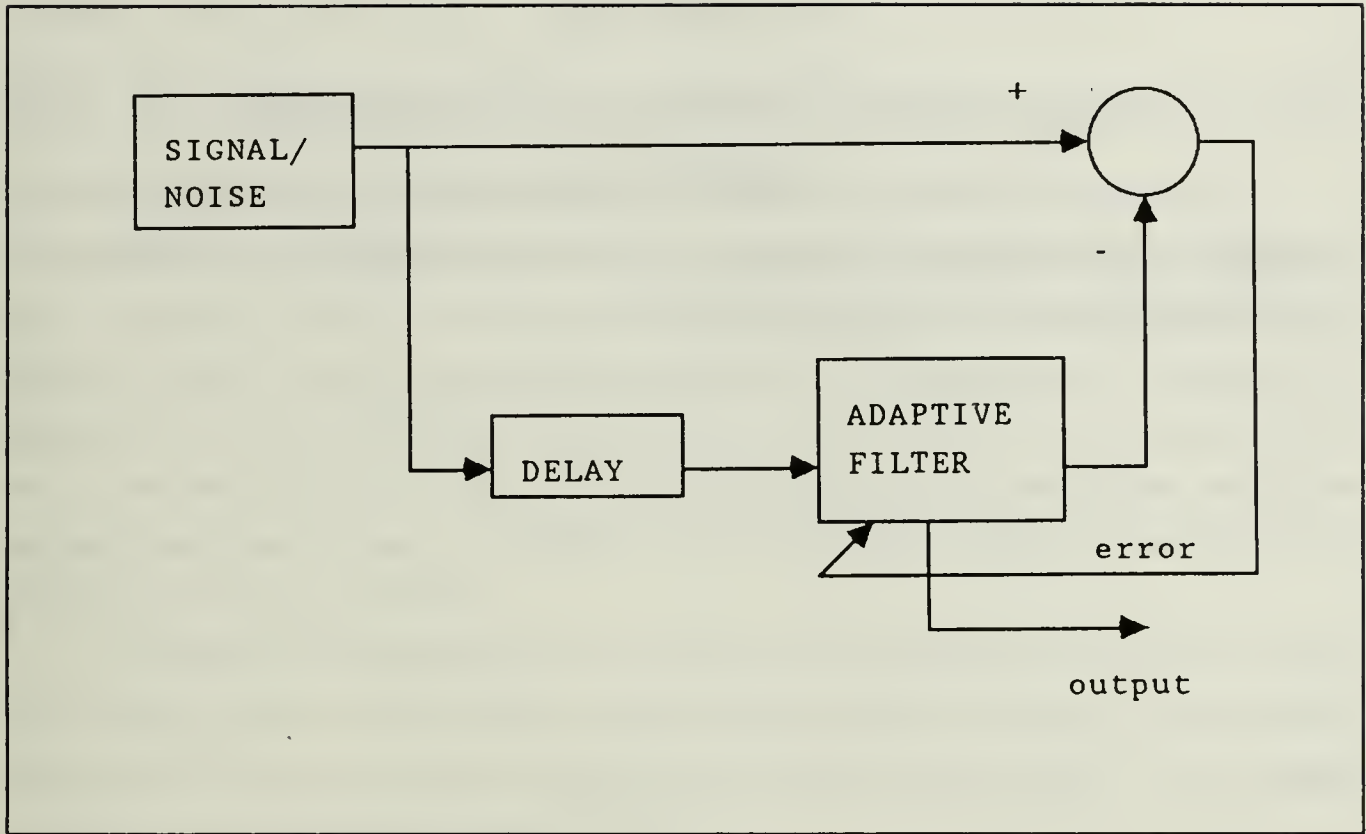


Figure 8. Adaptive Line Enhancer

delay Δ is selected so that it falls between the correlation lengths. Since Δ is longer than the effective correlation length of the broadband component, the delayed replica will be entirely uncorrelated with the broadband part of the main signal. The adaptive filter will not be able to respond to this component. On the other hand, since Δ is shorter than the correlation length of the narrowband component, the delayed replica that appears in the secondary input will be correlated with narrowband part of the main signal and the filter will respond to cancel it. Note that if Δ is selected to be longer than both correlation lengths, the secondary input will become uncorrelated with the primary input and the adaptive filter will turn itself off. In the opposite case, when the delay Δ is selected to be less than both correlation lengths, then both components of the secondary signal will be correlated with the primary signal, and therefore the adaptive filter will respond to cancel the primary $x(n)$ completely. The computational algorithm for the adaptive line enhancer is shown in the following three equations:

$$\hat{x}(n) = \sum_{m=0}^M h_m(n)y(n-m) = \sum_{m=0}^M h_m(n)x(n-m-\Delta) \quad (2.40)$$

$$e(n) = x(n) - \hat{x}(n) \quad (2.41)$$

$$h_m(n+1) = h_m(n) + 2\mu e(n) x(n-m-\Delta) \quad m = 0, 1, 2, \dots, M \quad (2.42)$$

For this model we also developed a computer simulation which incorporates our two dimensional LMS algorithm and it is provided in Appendix C. The results and conclusions will again be discussed in Chapter 4.

III. TWO DIMENSIONAL ADAPTIVE RECURSIVE LEAST SQUARES ALGORITHM

In the previous chapter, under the LMS algorithm, the available data samples were used in order to attempt to move the current estimate of the impulse response to the optimum value. This approach has the advantage of being simple to implement but carries with it the disadvantages that it can be slow to approach the optimal weight vector and, once close to it, will usually fluctuate around the optimal vector rather than actually converge to it due to the effects of approximations made in the estimate of the performance function gradient.

To overcome these difficulties, we examine another approach in this chapter which uses the input data in such a way as to ensure optimality at each step. This alternative algorithm is based on the exact minimization of the least square criteria. This algorithm is known as recursive least squares (RLS).

A. ONE DIMENSIONAL RECURSIVE LEAST SQUARES

As in the LMS case, the one dimensional RLS algorithms is developed in several different ways [Refs. 5, 8]. Orfanidis [Ref. 6] provides a derivation which we will consider prior to our extension to two dimensions. The tapped delay line shown in Figure 9 on page 22 will provide the reference for the following discussion. We begin by replacing the LMS estimation criteria of $MSE = E[e^2]$ by

$$MSE = \sum_{k=0}^n e^2(k) \quad k = 0, 1, \dots, n \quad (3.2)$$

where

$$e(k) = x(k) - \hat{x}(k) \quad (3.3)$$

and $\hat{x}(k)$ is the estimate of $x(k)$ produced by the Mth-order Wiener filter

$$\hat{x} = \sum_{m=0}^M h(m) y(k-m) \quad (3.4)$$

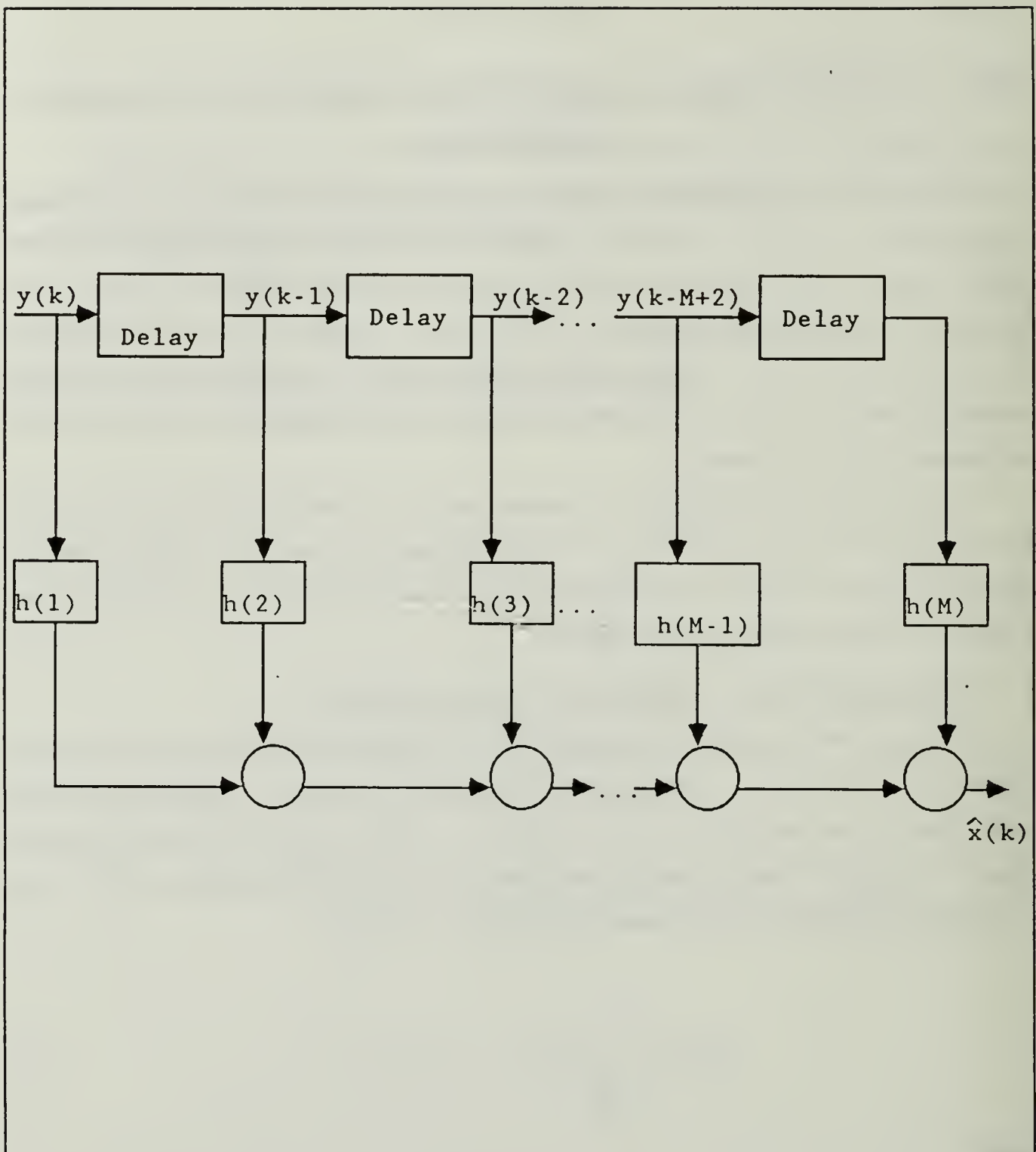


Figure 9. One Dimensional RLS Reference (TDL)

Substituting equation (3.2) into (3.1) and setting the derivative with respect to \mathbf{h} to zero we find the least-squares analogs of the orthogonality equations

$$\frac{\partial MSE}{\partial \mathbf{h}} = -2 \sum_{k=0}^n e(k) \mathbf{y}(k) = 0 \quad (3.5)$$

which may be rewritten in their normal equation form as follows

$$\sum_{k=0}^n (x(k) - \mathbf{h}^T \mathbf{y}(k)) \mathbf{y}(k) = 0 \quad (3.6)$$

$$\left[\sum_{k=0}^n \mathbf{y}(k) \mathbf{y}(k)^T \right] \mathbf{h} = \sum_{k=0}^n x(k) \mathbf{y}(k) \quad (3.7)$$

Defining the quantities

$$R(n) = \sum_{k=0}^n \mathbf{y}(k) \mathbf{y}(k)^T \quad (3.8)$$

$$\mathbf{r}(n) = \sum_{k=0}^n x(k) \mathbf{y}(k) \quad (3.9)$$

we can then write the normal equation as $R(n) \mathbf{h} = \mathbf{r}(n)$, with solution $\mathbf{h} = R(n)^{-1} \mathbf{r}(n)$. Note that the n -dependence of $R(n)$ and $\mathbf{r}(n)$ depend on the current time n , therefore,

$$\mathbf{h}(n) = R(n)^{-1} \mathbf{r}(n) = P(n) \mathbf{r}(n) \quad (3.10)$$

where $P(n) = R(n)^{-1}$. These are the least squares analogs of the ordinary Wiener solution, with $R(n)$ and $\mathbf{r}(n)$ playing the role of the covariance matrix $E[\mathbf{y}(n)\mathbf{y}(n)^T]$ and cross-correlation vector $E[x(n)\mathbf{y}(n)]$, respectively. The RLS algorithm is obtained by writing equation (3.10) recursively in n and then using the following matrix inversion lemma [Ref. 5]

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(DA^{-1}B + C^{-1})^{-1}DA^{-1} \quad (3.11)$$

we get the update equation for the P matrix

$$P(n) = P(n-1) - \frac{P(n-1)\mathbf{y}(n)\mathbf{y}(n)^T P(n-1)}{1 + \mathbf{y}(n)^T P(n-1)\mathbf{y}(n)}. \quad (3.12)$$

Using the quantities in equations (3.8) and (3.9) to satisfy the recursions yields

$$R(n) = R(n-1) + y(n)y(n)^T \quad (3.13)$$

$$r(n) = r(n-1) + x(n)y(n) \quad (3.14)$$

and substituting equations (3.12) and (3.14) into (3.10) after some mathematical manipulations we find

$$h(n) = h(n-1) + P(n) y(n) e(n) \quad (3.15)$$

which differs from the LMS algorithm by the presence of the $P(n)$, vice μ , in front of the weight correction term. Since $P(n) = R(n)^{-1}$ is a measure of the covariance matrix $E[y(n)y(n)^T]$, the presence of $R(n)^{-1}$ makes the RLS algorithm behave like Newton's method, and hence has fast convergence properties.

B. TWO DIMENSIONAL RECURSIVE LEAST SQUARES

Although it is discussed briefly by Wellstead and Caldas Pintos [Ref. 14], limited information is available in the open literature in this area. In order to develop a two dimensional recursive least square (RLS) algorithm we will extend the one dimensional adaptive algorithm developed in the previous section in a method similar to that used for the LMS algorithm.

As in Chapter 2, using Figure 10 on page 25 as a reference we see that the basic filter has two input images. The primary two dimensional input array, D , is the ideal image plus additive noise. The reference image X is the noise array. Each array is of dimension M by M . The filter mask, W , is N by N .

The one dimensional RLS algorithm, equation (3.15), is the same as the one dimensional LMS algorithm with the exception that $P(n)$ replaces μ . Therefore, if a method of developing a P matrix for the two dimensional case can be devised we can then use an algorithm similar to the two dimensional LMS algorithm to update the filter coefficients. First, let

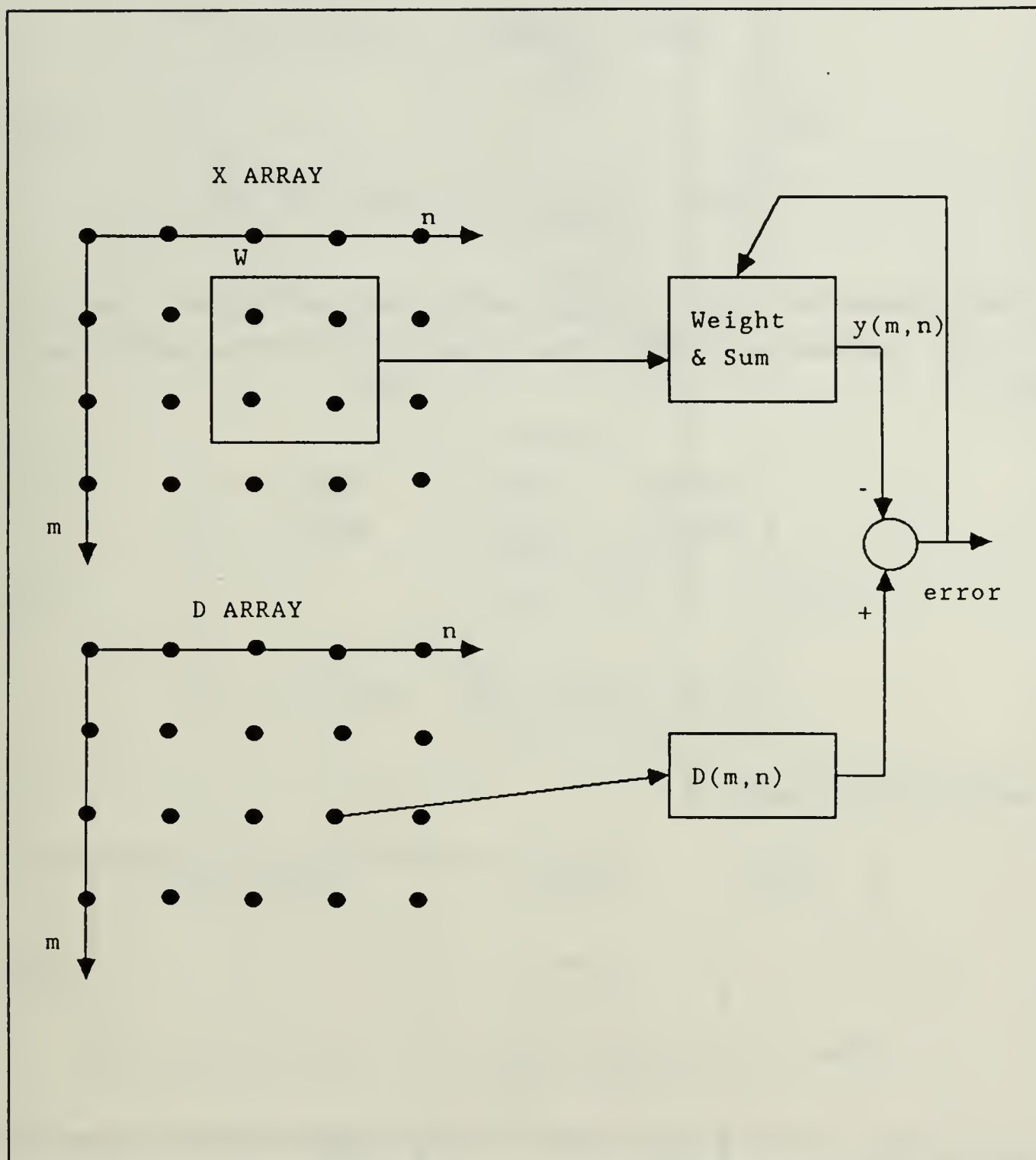


Figure 10. Two Dimensional RLS Reference

$$P_j = \begin{bmatrix} P_j(0,0) & P_j(0,1) & \cdots & P_j(0,N^2-1) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ P_j(N^2-1,0) & P_j(N^2-1,1) & \cdots & P_j(N^2-1,N^2-1) \end{bmatrix} \quad (3.16)$$

which represents the P matrix on the j th iteration where j is defined as $j = mM + n$. As in the two dimensional LMS algorithm, we let the filter coefficient matrix be given as

$$W_j = \begin{bmatrix} W_j(0,0) & W_j(0,1) & \cdots & W_j(0,N-1) \\ W_j(1,0) & W_j(1,1) & \cdots & W_j(1,N-1) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ W_j(N-1,0) & W_j(N-1,1) & \cdots & W_j(N-1,N-1) \end{bmatrix} \quad (3.17)$$

and the input data window as

$$X_j = \begin{bmatrix} X(m,n) & X(m,n-1) & \cdots & X(m,n-N+1) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ X(m-N+1,n) & X(m-N+1,n-1) & \cdots & X(m-N+1,n-N+1) \end{bmatrix} \quad (3.18)$$

In order to establish an update equation for the P matrix in which each element will have the proper dimensions, we use equations (3.17) and (3.18) and make the following transformations; the filter mask, W_j , equation (3.17) is transformed into a vector defined as

$$WW_j = \begin{bmatrix} W_j(0,0) \\ W_j(0,1) \\ \cdot \\ \cdot \\ W_j(0,N-1) \\ W_j(1,0) \\ W_j(1,1) \\ \cdot \\ \cdot \\ W_j(1,N-1) \\ \cdot \\ \cdot \\ W_j(N-1,0) \\ W_j(N-1,1) \\ \cdot \\ \cdot \\ W_j(N-1,N-1) \end{bmatrix} \quad (3.19)$$

and we transform the X_j matrix, equation (3.18) into

$$XX_j = \begin{bmatrix} X(m,n) \\ X(m,n-1) \\ \cdot \\ X(m,n-N+1) \\ \cdot \\ \cdot \\ X(m-N+1,n) \\ X(m-N+1,n-1) \\ \cdot \\ X(m-N+1,n-N+1) \end{bmatrix} \quad (3.20)$$

Using the quantities defined above, we can then write the P matrix update equation as

$$P_j = P_{j-1} - \frac{P_{j-1}(XX_j)(XX_j^T)P_{j-1}}{1 + (XX_j^T)P_{j-1}(XX_j)} \quad (3.21)$$

As discussed earlier, the one dimensional RLS weight updating algorithm differs from the LMS in that it contains the P matrix while the LMS algorithm contains μ . This must be considered in two dimensions since P is a matrix and μ is a scalar. Therefore using the previous equations, we can obtain the update to the filter coefficients via

$$WW_{j+1} = WW_j - (P_j)(e_j)(XX_j) \quad (3.22)$$

where e_j is given as

$$e_j = D(m,n) - y(m,n) \quad (3.23)$$

$y(m,n)$ is the convolution sum of the image in the reference input with the filter window

$$y(m,n) = \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} W_j(l,k) X_j(l,k) \quad (3.24)$$

and the value P_j is updated by equation (3.21).

The cost function of the RLS criterion could be modified to include a windowing of the input data as follows

$$MSE = \sum_{k=0}^n q^{n-k} e^2(k) \quad (3.25)$$

This modification results in the following modified P matrix update:

$$P_j = \frac{1}{q} \left[P_{j-1} - \frac{P_{j-1}(XX_j)(XX_j^T)P_{j-1}}{1 + (XX_j^T)P_{j-1}(XX_j)} \right] \quad (3.26)$$

where q , the averaging or "forgetting factor" is a positive constant. It is usually chosen to be slightly less than one, thereby diminishing the contribution of the "older" data. The problem of data nonstationarity is the main reason for introducing this type of weighting factor.[Ref. 6] It should be noted that the usual RLS algorithm is attained when $q = 1$ and that no changes in the amount of computation is required.

C. IMPLEMENTATION

1. System Identification

Following the mathematical development of the two dimensional recursive least squares algorithm, we implemented and tested it as was previously done with the two dimensional least mean square algorithm. Our initial testing was done in a system identification model as shown in Figure 11. As in the two dimensional LMS case, we used a known filter with the following output equation

$$\begin{aligned} d(m,n) = & .4W(m,n) + .6W(m-1,n) \\ & - .3W(m,n-1) + .2W(m-1,n-1) \end{aligned} \quad (3.27)$$

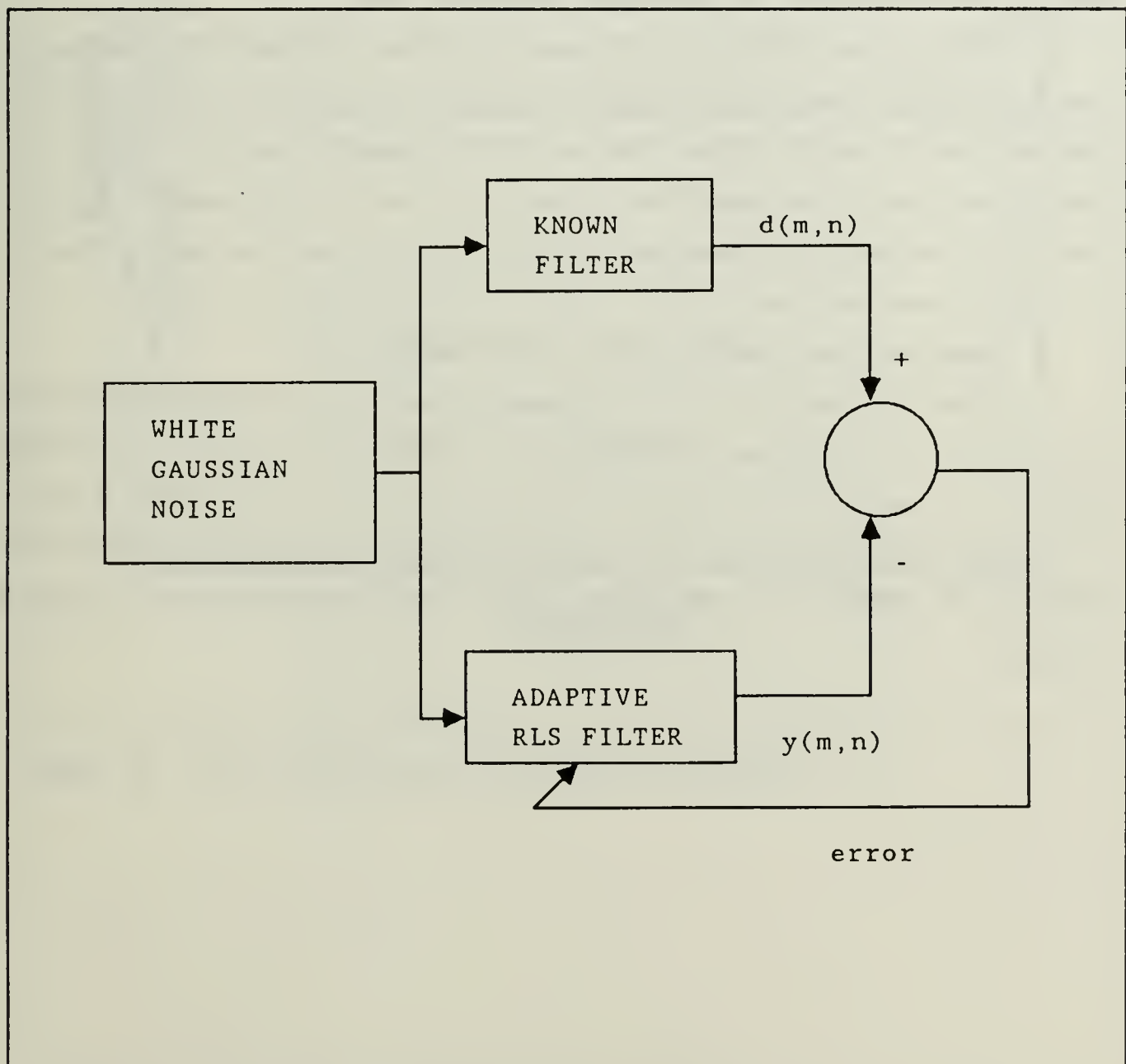


Figure 11. Two Dimensional RLS System Identification

The two dimensional adaptive recursive least square filter output was given as

$$\begin{aligned} y(m,n) = & A0 W(m,n) + A1 W(m-1,n) \\ & + A2 W(m,n-1) + A3 W(m-1,n-1) \end{aligned} \quad (3.28)$$

The error signal was generated by taking the difference between the desired (known) signal, $d(m,n)$, and the filter output, $y(m,n)$. By using equations (3.21) and (3.22) to update the P matrix and the filter coefficients, we caused the filter weights to move toward their optimum values and the error signal to approach zero.

The computer program for this model is provided in Appendix D. For this system, a 32x32 white gaussian noise matrix was generated for the input to both the known filter and the RLS adaptive filter. The rate of convergence is shown in Figure 12 on page 31 and Figure 13 on page 32 and as can be seen after approximately 70 iterations each of the coefficients had converged to within 10^{-3} of the actual value and the error was 10^{-5} .

2. Adaptive Noise Canceler/Adaptive Line Enhancer

These two systems were both discussed in Chapter 2 with regard to implementation of the two dimensional LMS algorithm. In order to further test the RLS algorithm we also implemented it within the noise canceler and the adaptive line enhancer. The computer programs which were used for the simulation are given in Appendix E and Appendix F, respectively. The simulation results and discussion will be provided in Chapter 4.

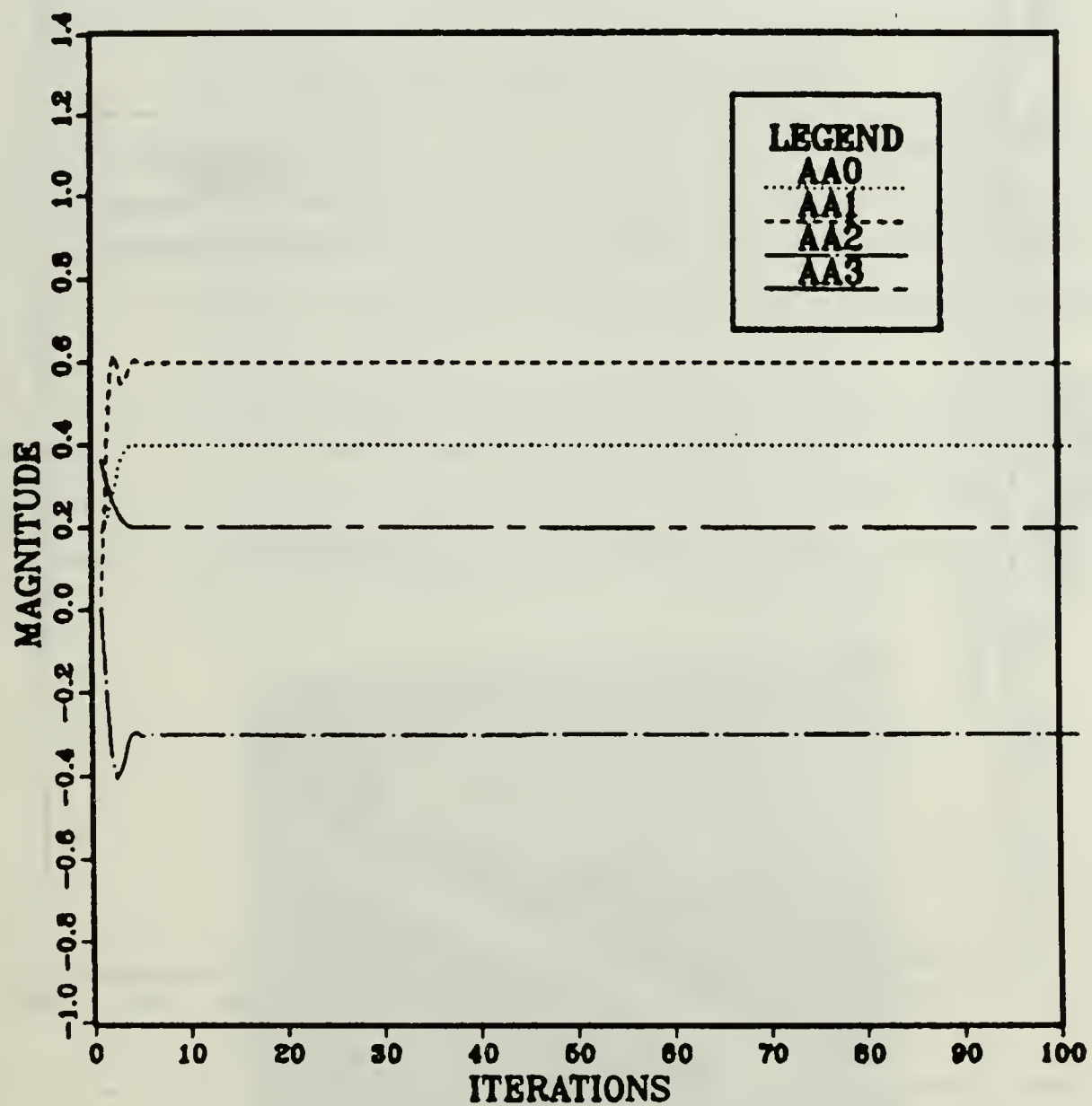


Figure 12. RLS System Identification Rate of Convergence

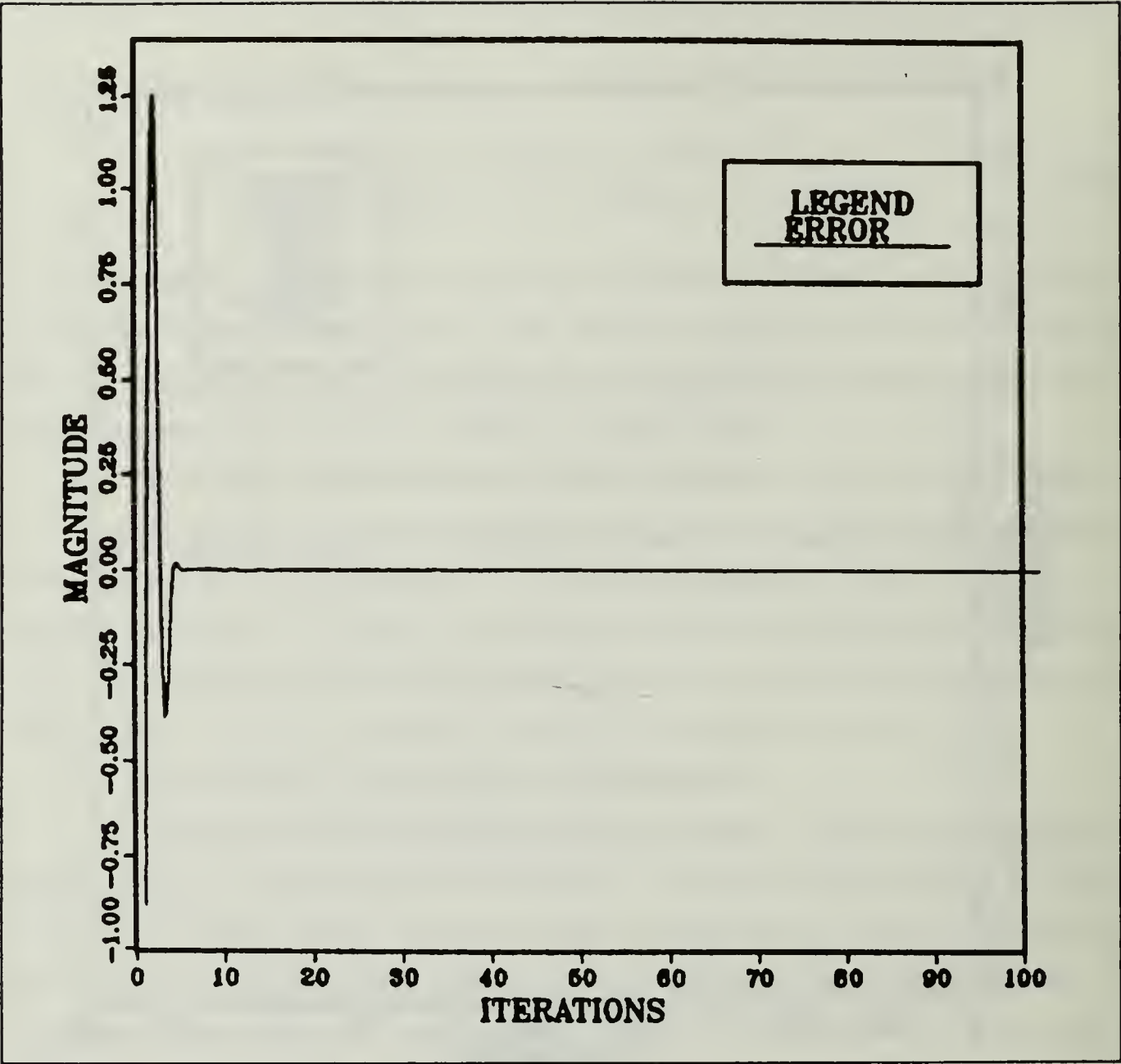


Figure 13. RLS System Identification Rate of Convergence

IV. RESULTS AND CONCLUSIONS

In this chapter, we present the experimental results from our implementation of the two dimensional LMS algorithm derived in Chapter II and the two dimensional RLS algorithm derived in Chapter III. The two algorithms were used in a noise canceler and an adaptive line enhancer, as discussed in Chapter II, to solve an image restoration problem.

Figure 14 shows the original image of a house which is comprised of 128 by 128 eight bit pixels. The image has a mean value of 131.68 and a variance of 3194.4. In Figure 15 on page 34, we have corrupted the original image with additive white gaussian noise (zero mean and variance 1600).

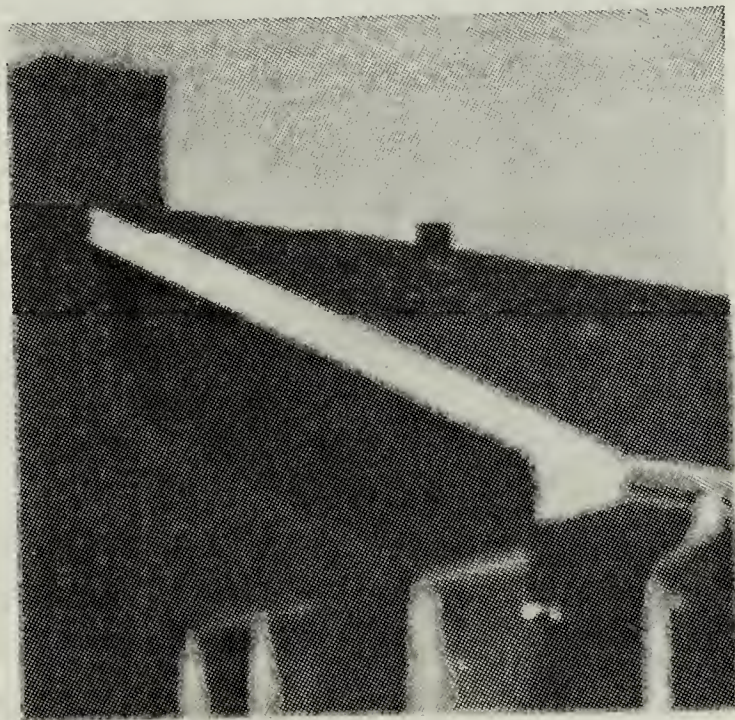


Figure 14. Original Image

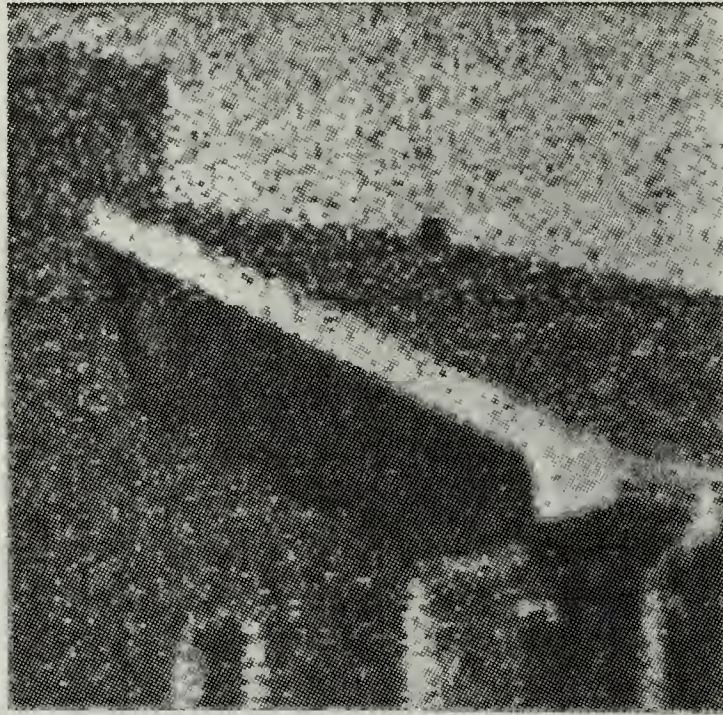


Figure 15. Image plus Additive Noise

A. NOISE CANCELER RESULTS

In order to solve our image restoration problem, we initially implemented both the LMS and the RLS algorithms in a noise canceler using a two by two filter matrix and a value of 70×10^{-9} for μ in the LMS algorithm.

Figure 16 on page 35 shows the output when the LMS algorithm is processed through the image for one pass: this results in 16,384 updates of the filter corresponding to the 16,384 pixels. Figure 17 on page 36 is the results following two passes through the image. No significant improvement was realized with more than two passes.

Implementing a two by two RLS adaptive filter in the noise canceler produced very favorable results after one pass through the image. These results are shown in Figure 18 on page 36 and compare well with the original image and the LMS output after two passes.

For the LMS algorithm our best results were achieved with a value of 70×10^{-9} for μ , however in order to demonstrate the effect of changing this value Figure 19 on page 37 represents the output with $\mu = 35 \times 10^{-9}$ and Figure 20 on page 37 is produced by a spatially varying normalized μ as discussed in Chapter II. It can be seen that for various

values of μ , different features within the image were restored at different levels. In Figure 16 on page 35, the edges and more detailed segments of the image were restored better than in Figure 19 on page 37, however areas of similar contrast were not as well restored. When using the normalized μ the mean value more closely approached the original mean, however the variance was reduced.

Finally, increasing the number of coefficients, using a three by three filter matrix in the LMS algorithm we obtained the results shown in Figure 21 on page 38 in one pass. This filter showed no significant improvement in the variance compared to the two by two filter; however, it did improve with respect to the mean.

Table 1 on page 38 shows the restoration results comparing the mean and variance of the various outputs with that of the original image and the image plus noise.

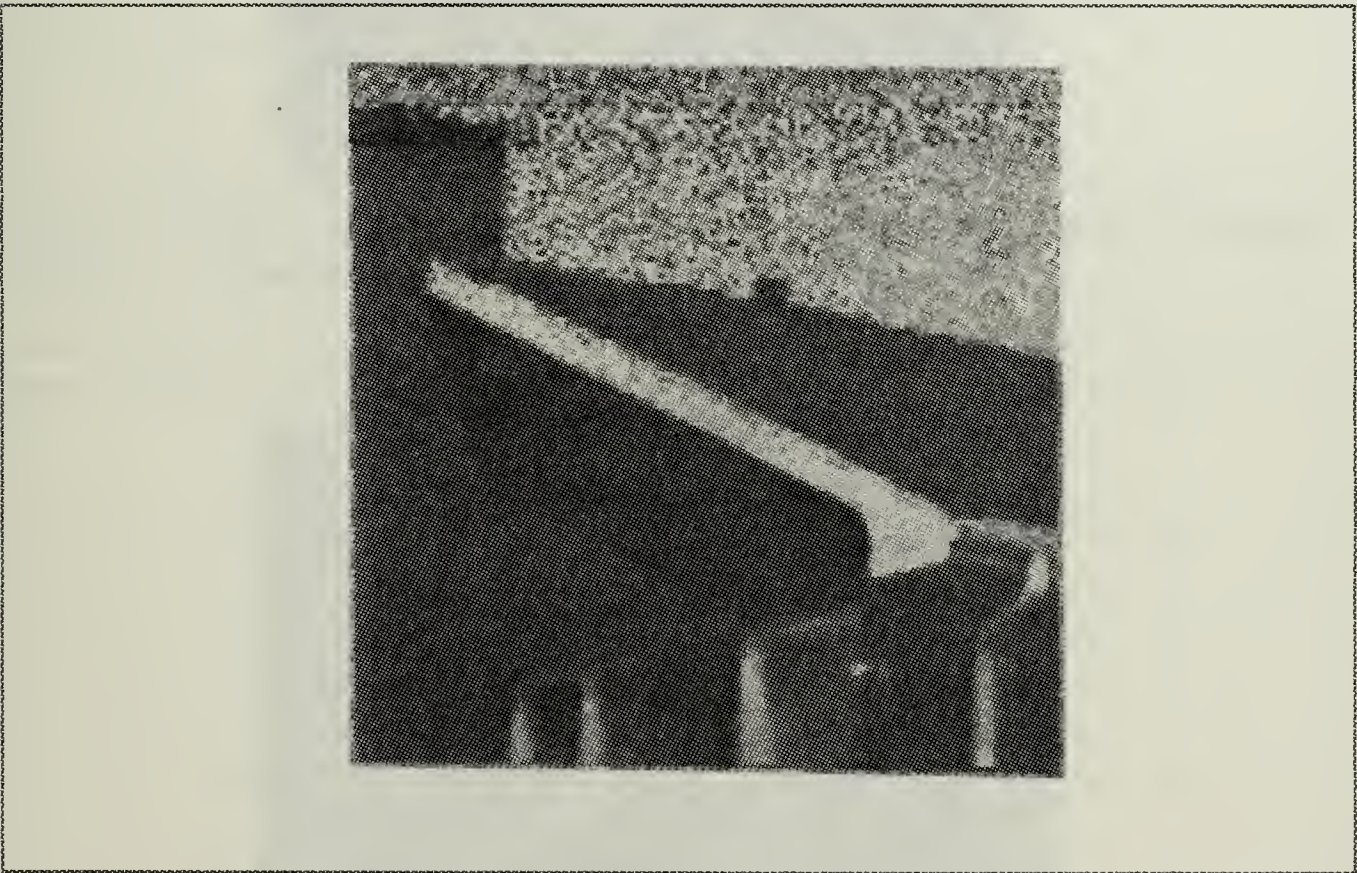


Figure 16. Restored Image: Noise Canceler/LMS (One pass)

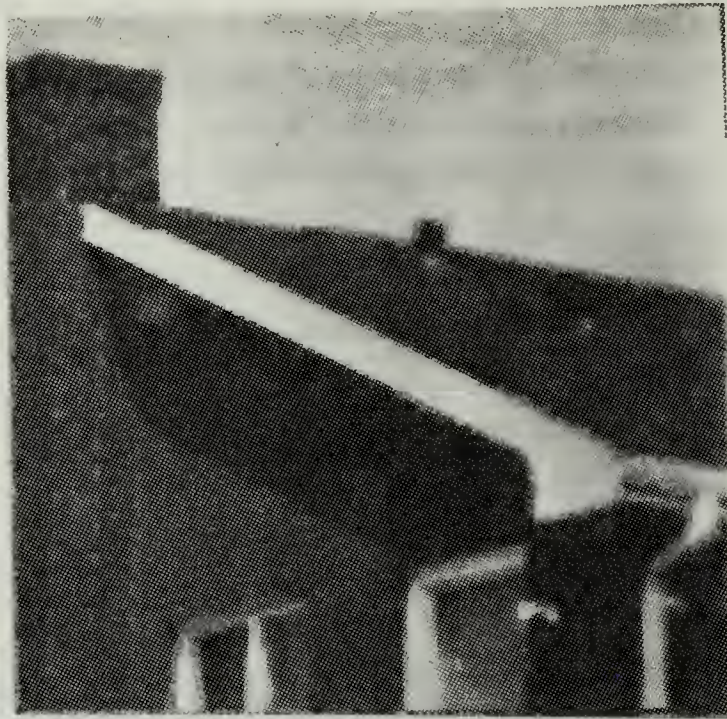


Figure 17. Restored Image: Noise Canceler/LMS (Two passes)

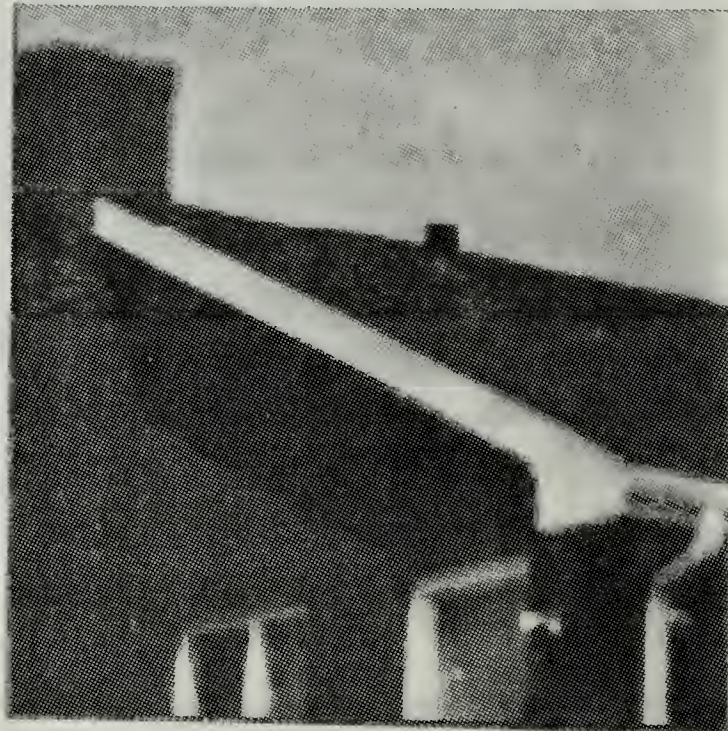


Figure 18. Restored Image: Noise Canceler/RLS (One pass)

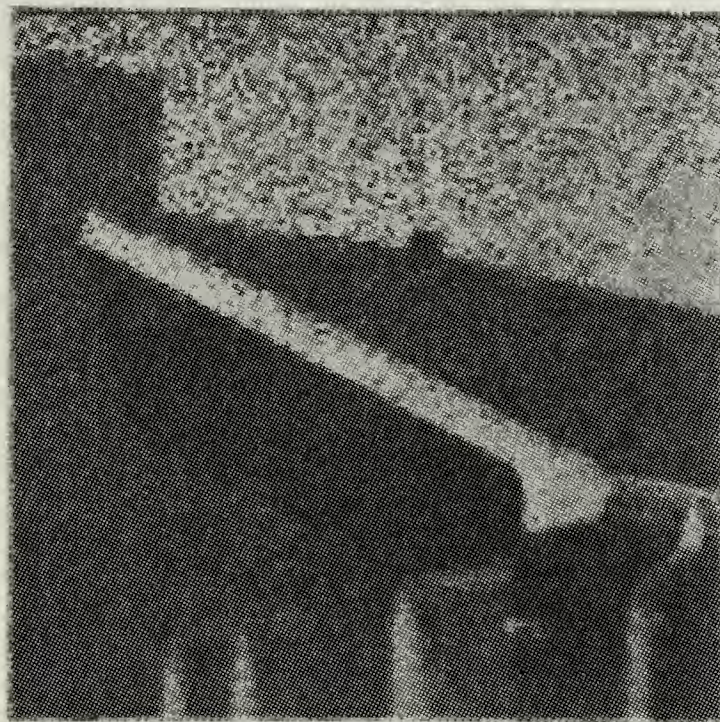


Figure 19. Restored Image: Noise Canceler/LMS (different μ)

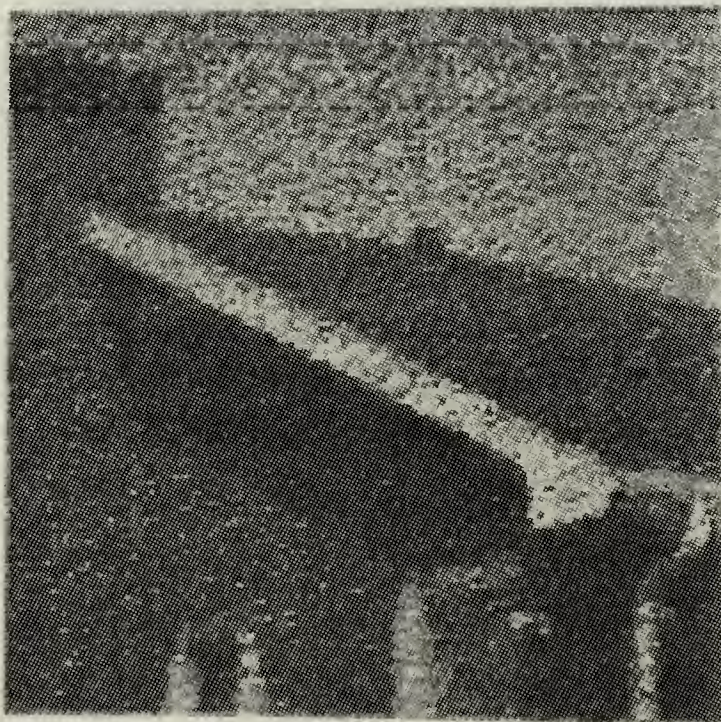


Figure 20. Restored Image: Noise Canceler/LMS (normalized μ)

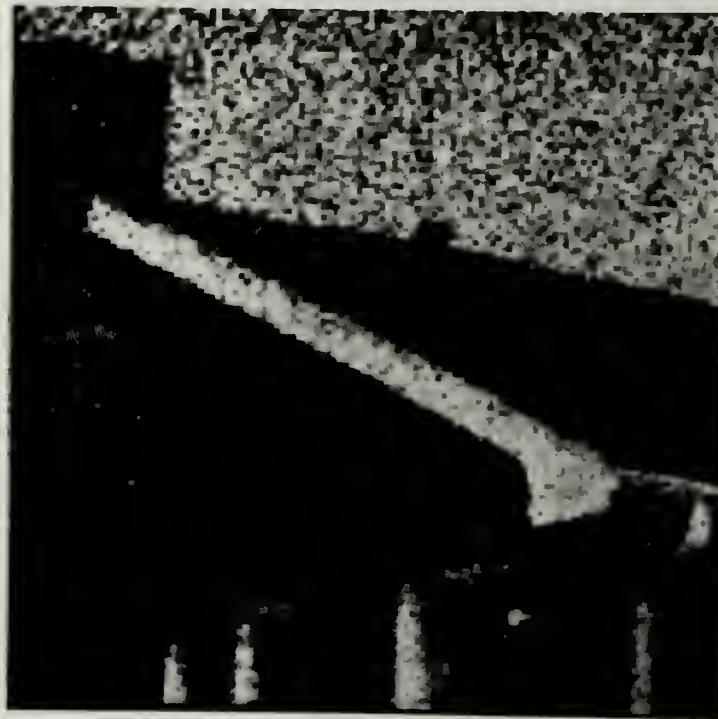


Figure 21. Restored Image: Noise Canceler/LMS (3 by 3)

Table 1. NOISE CANCELER RESULTS

TYPE OF FILTER	MEAN	VARIANCE
Original Image	131.68	3194.4
Image plus Noise	134.25	1647.7
One pass 2x2 LMS	114.52	2034.3
Two pass 2x2 LMS	136.78	3711.8
One pass 2x2 RLS	139.04	3607.7
One pass LMS(different mu)	120.12	1940.4
One pass LMS(normalized mu)	126.53	1500.7
One pass 3x3 LMS	119.13	2017.3

B. ADAPTIVE LINE ENHANCER RESULTS

We next implemented our two algorithms in an adaptive line enhancer. In order to obtain a reference input from the primary input, we used a two dimensional delay operator of $(m-1, n)$. As in the noise canceler we again used a two by two filter matrix and this produced the results shown in Figure 22 on page 39 and Figure 23 on page 40 for the one pass LMS and the one pass RLS, respectively. No significant improvement was noted in the two pass LMS compared to the one pass LMS.

Table 2 on page 40 shows the comparison of each output mean and variance with the mean and variance of the original image and the image plus noise.

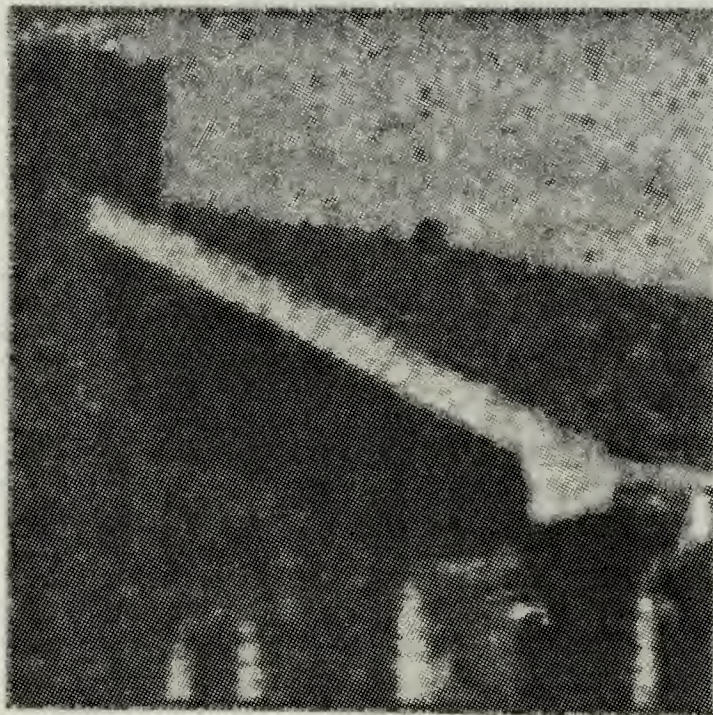


Figure 22. Restored Image: Adaptive Line Enhancer/LMS (One pass)

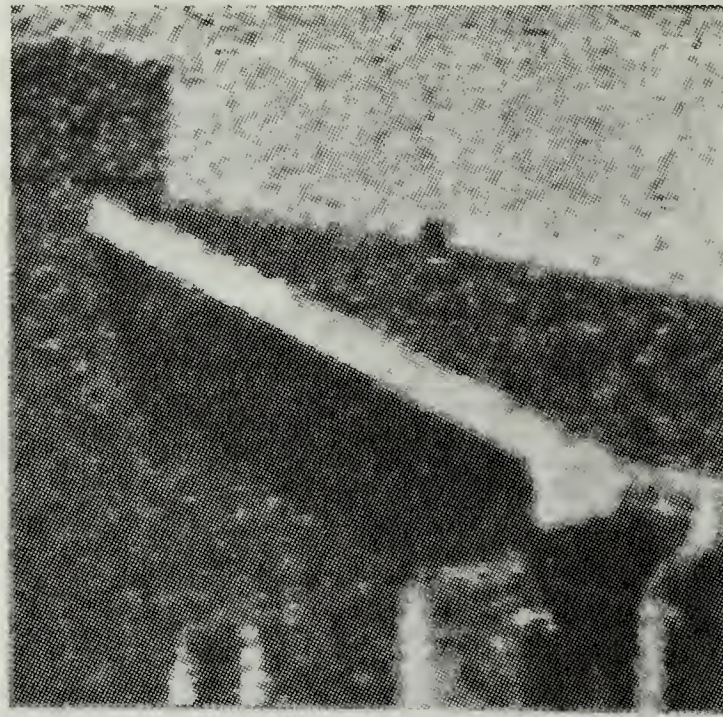


Figure 23. Restored Image: Adaptive Line Enhancer/RLS (One pass)

Table 2. ADAPTIVE LINE ENHANCER RESULTS

TYPE OF FILTER	MEAN	VARIANCE
Original Image	131.68	3194.4
Image plus Noise	134.25	1647.7
One pass 2x2 LMS	125.03	2521.8
Two pass 2x2 LMS	126.94	2529.9
One pass 2x2 RLS	139.69	2124.2

C. CONCLUSIONS

In this thesis we have extended a one dimensional LMS and a one dimensional RLS adaptive algorithm to two dimensions. As we examine the results we see that many of the one dimensional comparisons between LMS and RLS are applicable to two dimensions:

1. the rate of convergence of the RLS algorithm is in general faster than that of the LMS algorithm by an order of magnitude
2. this superior performance of the RLS algorithm is attained at the expense of a large increase in the computational complexity

3. there are no approximations within the derivation of the RLS algorithm, therefore as the number of iterations approach infinity, the least squares estimate approaches the optimum Wiener value
4. in the RLS algorithm the correction which is applied is computed using all past data whereas the LMS algorithm uses only the instantaneous sample and the error signal; this is not necessarily an advantage in image processing.

The objectives of this thesis were successfully completed. Some suggestions for future work include: (i) to examine these two dimensional algorithms in other applications, applying them to areas other than image processing, (ii) to extend the two dimensional LMS and RLS algorithms to n-dimensions and implement them, (iii) to analyze the extension of these one dimensional algorithms to two dimensions in the frequency domain. Multidimensional digital signal processing is being applied in many areas today, however the potential for future growth and applications appears unlimited.

APPENDIX A. LMS SYSTEM IDENTIFICATION PROGRAM

```

C      SYSTEM IDENTIFICATION OF AN ADAPTIVE TWO DIMENSIONAL
C      LMS FILTER VS. A KNOWN TWO DIMENSIONAL FILTER
C
      REAL MU
      DIMENSION A0(0:32,0:32),A1(0:32,0:32),A2(0:32,0:32),
*      A3(0:32,0:32),E(0:32,0:32),D(0:32,0:32),W(-1:32,-1:32),
*      Y(0:32,0:32)
C
C      *****VARIABLE DEFINITIONS*****
C      A=NOISE AMPLITUDE
C      S=NOISE VARIANCE
C      AM=NOISE MEAN
C      IX=SEED
C      A0,A1,A2,A3=FILTER COEFFICIENTS
C      MU=SCALING FACTOR WITHIN THE LMS ALGORITHM
C      W=WHITE GAUSSIAN NOISE GENERATED BY SUBROUTINE
C      Y=FILTER OUTPUT
C      D=DESIRED OUTPUT
C      E=ERROR(D - Y)
C
C      *****INITIAL VALUES*****
      A=1.
      MU=.01
      A0(0,0)=0.
      A1(0,0)=0.
      A2(0,0)=0.
      A3(0,0)=0.
      S=1.
      AM=0.
      IX=65539
C
C      *****OPEN FILE AND FILL INPUT MATRIX WITH WHITE
      GAUSSIAN NOISE*****
      OPEN (UNIT=80,FILE='SYSID2 DATA',STATUS='NEW')
      DO 10 M=-1,31
        DO 20 N=-1,31
          IF((M.LT.0).OR.(N.LT.0))THEN
            W(M,N)=0.
          ELSE
            CALL GAUSS(IX,S,AM,V)
            W(M,N)=V*A
          ENDIF
        CONTINUE
      CONTINUE
C
C      *****COMPUTE THE UPDATED FILTER COEFFICIENTS
      USING THE TWO DIMENSIONAL LMS ALGORITHM*****
      DO 40 K=0,31
        DO 50 J=0,31
          D(K,J)=.4*W(K,J)+.6*W(K-1,J)-.3*W(K,J-1)+.2*W(K-1,J-1)
          Y(K,J)=A0(K,J)*W(K,J)+A1(K,J)*W(K-1,J)+A2(K,J)*W(K,J-1)+

```



```

*      A3(K,J)*W(K-1,J-1)
      E(K,J)=D(K,J)-Y(K,J)
      A0(K,J+1)=A0(K,J)+MU*E(K,J)*W(K,J)
      A1(K,J+1)=A1(K,J)+MU*E(K,J)*W(K-1,J)
      A2(K,J+1)=A2(K,J)+MU*E(K,J)*W(K,J-1)
      A3(K,J+1)=A3(K,J)+MU*E(K,J)*W(K-1,J-1)
50    CONTINUE
      A0(K+1,0)=A0(K,J)
      A1(K+1,0)=A1(K,J)
      A2(K+1,0)=A2(K,J)
      A3(K+1,0)=A3(K,J)
40    CONTINUE
C
C      *****OUTPUT RESULTS*****
      L=0
      DO 60 K=0,31
        DO 70 J=0,31
          L=L+1
          PRINT 15, L,E(K,J),A0(K,J),A1(K,J),A2(K,J),A3(K,J)
          WRITE (80,15) L,E(K,J),A0(K,J),A1(K,J),A2(K,J),A3(K,J)
15      FORMAT (' ',1X,I4,2X,F8.5,2X,F8.5,2X,F8.5,2X,F8.5,
*            2X,F8.5)
70      CONTINUE
60    CONTINUE
      STOP
      END
C
      SUBROUTINE GAUSS(IX,S,AM,V)
      A=0.0
      DO 50 I=1,12
        CALL RANDU(IX,IY,Y)
        IX=IY
50      A=A+Y
      V=(A-6.0)*S+AM
      RETURN
      END
C
C      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*65539
      IF(IY)5,6,6
5      IY=IY+2147483647+1
6      YFL=IY
      YFL=YFL*.4656613E-9
      RETURN
      END

```

APPENDIX B. LMS ALGORITHM IMPLEMENTED IN A NOISE CANCELER

```

C      THIS IS A VAX/VMS FORTRAN PROGRAM THAT IMPLEMENTS A TWO
C      DIMENSIONAL 2X2 ADAPTIVE LMS FILTER WITHIN A NOISE
C      CANCELER
C
      INTEGER M,N,K,J
      BYTE B(0:127),BYTEE(0:127),BYTEU(0:127),
      INTEGER*4 INTE(0:127,0:127),INTU(0:127,0:127),
*      IE,IU
      REAL*4 MU,A,FMINE,FMAXE,FMINU,FMAXU
      REAL*4 AA(0:3),E(0:127,0:127),U(0:127,0:127)
      REAL*4 W(-1:127,-1:127),Y(0:127,0:127),IM(0:127,0:127)

C      *****VARIABLE DEFINITIONS*****
C      A=NOISE AMPLITUDE
C      S=NOISE VARIANCE
C      AM=NOISE MEAN
C      AA=FILTER COEFFICIENTS
C      IX=SEED
C      MU=SCALING FACTOR WITHIN THE LMS ALGORITHM
C      IM=INPUT IMAGE
C      W=WHITE GAUSSIAN NOISE GENERATED BY SUBROUTINE
C      U=SIGNAL PLUS NOISE(IM + W)
C      Y=FILTER OUTPUT
C      E=ERROR(U - Y)
C      FMINE,FMAXE,FMINU,FAMXU=PARAMETERS
C      TO BE USED TO CONVERT DECIMAL DATA TO BYTE DATA
C
C      *****INITIAL VALUES*****
      A=1.
      MU=35.E-9
      AA(0)=0.
      AA(1)=0.
      AA(2)=0.
      AA(3)=0.
      S=40.
      AM=0.
      IX=65539
      FMINE=1.E+10
      FMAXE=-1.E+10
      FMINU=1.E+10
      FMAXU=-1.E+10

C      *****OPEN AN IMAGE FILE, CONVERT THE BYTE DATA INTO INTEGERS
C      AND THEN PLACE THESE VALUES IN A MATRIX*****
C      OPEN (UNIT=1, NAME ='HOUS1G.DAT', TYPE ='OLD', ACCESS =
*      'DIRECT', RECORDSIZE=32, MAXREC=128)
      DO 100 M=0,127
        READ(1'M+1) (B(J),J=0,127)
        DO 110 N=0,127

```



```

        IF(B(N).LT.0)THEN
        IM(M,N)=B(N)+256
        ELSE
        IM(M,N)=B(N)
        ENDIF
110    CONTINUE
100    CONTINUE
C
C    *****ADD WHITE GAUSSIAN NOISE TO THE IMAGE AND SET THE
C    VALUES OUTSIDE THE IMAGE TO ZERO*****
DO 10 M=-1,127
    DO 20 N=-1,127
        IF ((M.LT.0).OR.(N.LT.0))THEN
            W(M,N)=0.
            IM(M,N)=0.
        ELSE
            CALL GAUSS(IX,S,AM,V)
            W(M,N)=V*A
        ENDIF
        U(M,N)=IM(M,N)+W(M,N)
20    CONTINUE
10    CONTINUE
C
C    *****USE THE LMS ADAPTIVE ALGORITHM TO UPDATE
C    THE FILTER COEFICIENTS*****
DO 40 K=0,127
    DO 50 J=0,127
        Y(K,J)=AA(0)*W(K,J)+AA(1)*W(K-1,J)+
*        AA(2)*W(K,J-1)+AA(3)*W(K-1,J-1)
        E(K,J)=U(K,J)-Y(K,J)
        AA(0)=AA(0)+MU*E(K,J)*W(K,J)
        AA(1)=AA(1)+MU*E(K,J)*W(K-1,J)
        AA(2)=AA(2)+MU*E(K,J)*W(K,J-1)
        AA(3)=AA(3)+MU*E(K,J)*W(K-1,J-1)
50    CONTINUE
40    CONTINUE
C
C    *****CHANGE SIGNAL PLUS NOISE AND ERROR
C    OUTPUT INTO BYTE DATA AND THEN WRITE
C    TO A FILE*****
OPEN (UNIT=2,NAME='ERROR.DAT',TYPE='NEW',ACCESS=
* 'DIRECT',RECORDSIZE=32,MAXREC=128)
OPEN (UNIT=3,NAME='SIGNOISE.DAT',TYPE='NEW',ACCESS=
* 'DIRECT',RECORDSIZE=32,MAXREC=128)
DO 500 I=0,127
    DO 550 J=0,127
        IF(E(I,J).LT.FMINE)THEN
            FMINE=E(I,J)
        ENDIF
        IF(E(I,J).GT.FMAXE)THEN
            FMAXE=E(I,J)
        ENDIF
550    CONTINUE
500    CONTINUE
    IF (FMINE.LT.0) THEN
        FMAXE=FMAXE-FMINE

```

```

ENDIF
DO 600 I=0,127
  DO 650 J=0,127
    IF(FMINE.LT.0)THEN
      E(I,J)=E(I,J)-FMINE
      E(I,J)=E(I,J)*255./FMAXE
    ELSE
      E(I,J)=(E(I,J)=FMINE)*255./FMAXE
    ENDIF
    IE=NINT(E(I,J))
    IF(IE.GT.127) THEN
      IE=IE-256
    ENDIF
    BYTEE(J)-IE
650    CONTINUE
    WRITE (2'I+1) (BYTEE(N),N=0,127)
600  CONTINUE
  DO 700 I=0,127
    DO 750 J=0,127
      IF(U(I,J).LT.FMINU)THEN
        FMINU=U(I,J)
      ENDIF
      IF(U(I,J).GT.FMAXU)THEN
        FMAXU=U(I,J)
      ENDIF
750    CONTINUE
700  CONTINUE
    IF (FMINU.LT.0) THEN
      FMAXU=FMAXU-FMINU
    ENDIF
    DO 400 I=0,127
      DO 450 J=0,127
        IF(FMINU.LT.0)THEN
          U(I,J)=U(I,J)-FMINU
          U(I,J)=U(I,J)*255./FMAXU
        ELSE
          U(I,J)=(U(I,J)=FMINU)*255./FMAXU
        ENDIF
        IU=NINT(U(I,J))
        IF(IU.GT.127) THEN
          IU=IU-256
        ENDIF
        BYTEU(J)-IU
450    CONTINUE
        WRITE (3'I+1) (BYTEU(N),N=0,127)
400  CONTINUE
    CLOSE (UNIT=2)
    CLOSE (UNIT=3)
    STOP
    END
C
SUBROUTINE GAUSS(IX,S,AM,V)
AMP=0.0
DO 50 I=1,12
  RANDOM=RAN(IX)

```

```
50      AMP=AMP+RANDOM  
      CONTINUE  
      V=(AMP-6.0)*S+AM  
      RETURN  
      END
```

APPENDIX C. LMS ALGORITHM IMPLEMENTED IN AN ADAPTIVE LINE ENHANCER

```

C      THIS IS A VAX/VMS FORTRAN PROGRAM THAT IMPLEMENTS A TWO
C      DIMENSIONAL 2X2 ADAPTIVE LMS FILTER WITHIN AN ADAPTIVE
C      LINE ENHANCER
C
      INTEGER M,N,K,J
      BYTE B(0:127),BYTEY(0:127),BYTEU(0:127)
      INTEGER*4 INTY(0:127,0:127),INTU(0:127,0:127),IY,IU
      REAL*4 MU,A,FMINY,FMAXY,FMINU,FMAXU
      REAL*4 AA(0:3),E(0:127,0:127),U(0:127,0:127)
      REAL*4 W(-1:127,-1:127),Y(0:127,0:127),IM(0:127,0:127)
C
C      *****VARIABLE DEFINITIONS*****
C      A=NOISE AMPLITUDE
C      S=NOISE VARIANCE
C      AM=NOISE MEAN
C      AA=FILTER COEFFICIENTS
C      IX=SEED
C      MU=SCALING FACTOR WITHIN THE LMS ALGORITHM
C      IM=INPUT IMAGE
C      W=WHITE GAUSSIAN NOISE GENERATED BY SUBROUTINE
C      U=SIGNAL PLUS NOISE(IM + W)
C      Y=FILTER OUTPUT
C      WW=DELAYED VERSION OF SIGNAL PLUS NOISE(U)
C      E=ERROR(U - Y)
C      FMINY,FMAXY,FMINU,FAMXU=PARAMETERS
C      TO BE USED TO CONVERT DECIMAL DATA TO BYTE DATA
C
C      *****INITIAL VALUES*****
      A=1.
      MU=35. E-9
      AA(0)=0.
      AA(1)=0.
      AA(2)=0.
      AA(3)=0.
      S=40.
      AM=0.
      IX=65539
      FMINU=1. E+10
      FMAXU=-1. E+10
      FMINY=1. E+10
      FMAXY=-1. E+10
C
C      *****OPEN AN IMAGE FILE, CONVERT THE BYTE DATA INTO INTEGERS
C      AND THEN PLACE THESE VALUES IN A MATRIX*****
      OPEN (UNIT=1, NAME ='HOUS1G.DAT', TYPE ='OLD', ACCESS =
*      'DIRECT', RECORDSIZE=32, MAXREC=128)
      DO 100 M=0,127
        READ(1'M+1) (B(J),J=0,127)
      DO 110 N=0,127

```

```

        IF(B(N).LT.0)THEN
        IM(M,N)=B(N)+256
        ELSE
        IM(M,N)=B(N)
        ENDIF
110      CONTINUE
100     CONTINUE
C
C      *****ADD WHITE GAUSSIAN NOISE TO THE IMAGE AND SET THE
C      VALUES OUTSIDE THE IMAGE TO ZERO*****
DO 10 M=-1,127
    DO 20 N=-1,127
        IF ((M.LT.0).OR.(N.LT.0))THEN
            W(M,N)=0.
            IM(M,N)=0.
        ELSE
            CALL GAUSS(IX,S,AM,V)
            W(M,N)=V*A
        ENDIF
        U(M,N)=IM(M,N)+W(M,N)
        WW(M,N)=U(M-1,N)
20      CONTINUE
10     CONTINUE
C
C      *****USE THE LMS ADAPTIVE ALGORITHM TO UPDATE
C      THE FILTER COEFICIENTS*****
DO 40 K=0,127
    DO 50 J=0,127
        Y(K,J)=AA(0)*WW(K,J)+AA(1)*WW(K-1,J)+
*          AA(2)*WW(K,J-1)+AA(3)*WW(K-1,J-1)
        E(K,J)=U(K,J)-Y(K,J)
        AA(0)=AA(0)+MU*E(K,J)*WW(K,J)
        AA(1)=AA(1)+MU*E(K,J)*WW(K-1,J)
        AA(2)=AA(2)+MU*E(K,J)*WW(K,J-1)
        AA(3)=AA(3)+MU*E(K,J)*WW(K-1,J-1)
50      CONTINUE
40     CONTINUE
C
C      *****CHANGE SIGNAL PLUS NOISE AND FILTER OUTPUT
C      INTO BYTE DATA AND THEN WRITE TO A FILE*****
OPEN (UNIT=3,NAME='SIGNOISE.DAT',TYPE='NEW',ACCESS=
*   'DIRECT',RECORDSIZE=32,MAXREC=128)
OPEN (UNIT=4,NAME='FILTERED.DAT',TYPE='NEW',ACCESS=
*   'DIRECT',RECORDSIZE=32,MAXREC=128)
DO 700 I=0,127
    DO 750 J=0,127
        IF(U(I,J).LT.FMINU)THEN
            FMINU=U(I,J)
        ENDIF
        IF(U(I,J).GT.FMAXU)THEN
            FMAXU=U(I,J)
        ENDIF
750      CONTINUE
700     CONTINUE
        IF (FMINU.LT.0) THEN
            FMAXU=FMAXU-FMINU

```



```

ENDIF
DO 400 I=0,127
  DO 450 J=0,127
    IF(FMINU. LT. 0) THEN
      U(I,J)=U(I,J)-FMINU
      U(I,J)=U(I,J)*255./FMAXU
    ELSE
      U(I,J)=(U(I,J)=FMINU)*255./FMAXU
    ENDIF
    IU=NINT(U(I,J))
    IF(IU. GT. 127) THEN
      IU=IU-256
    ENDIF
    BYTEU(J)-IU
450    CONTINUE
    WRITE (3'I+1) (BYTEU(N),N=0,127)
400  CONTINUE
DO 800 I=0,127
  DO 850 J=0,127
    IF(Y(I,J). LT. FMINY) THEN
      FMINY=Y(I,J)
    ENDIF
    IF(Y(I,J). GT. FMAXY) THEN
      FMAXY=Y(I,J)
    ENDIF
850    CONTINUE
800  CONTINUE
  IF (FMINY. LT. 0) THEN
    FMAXY=FMAXY-FMINY
  ENDIF
DO 900 I=0,127
  DO 950 J=0,127
    IF(FMINY. LT. 0) THEN
      Y(I,J)=Y(I,J)-FMINY
      Y(I,J)=Y(I,J)*255./FMAXY
    ELSE
      Y(I,J)=(Y(I,J)=FMINY)*255./FMAXY
    ENDIF
    IY=NINT(Y(I,J))
    IF(IY. GT. 127) THEN
      IY=IY-256
    ENDIF
    BYTEY(J)-IY
950    CONTINUE
    WRITE (4'I+1) (BYTEY(N),N=0,127)
900  CONTINUE
  CLOSE (UNIT=3)
  CLOSE (UNIT=4)
  STOP
  END

C
SUBROUTINE GAUSS(IX,S,AM,V)
AMP=0.0
DO 50 I=1,12
  RANDOM=RAN(IX)

```

```
50      AMP=AMP+RANDOM  
      CONTINUE  
      V=(AMP-6.0)*S+AM  
      RETURN  
      END
```

APPENDIX D. RLS SYSTEM IDENTIFICATION

```

C      SYSTEM IDENTIFICATION OF AN ADAPTIVE RLS 2X2 FILTER VERSUS
C      2X2 RLS FILTER VERSUS A A 2X2 FILTER WITH
C      KNOWN COEFFICIENTS
C
C      DIMENSION W(-1:32,-1:32),P(0:3,0:3),AA(0:3),X(0:3)
C
C      *****VARIABLE DEFINITIONS*****
C      A=NOISE AMPLITUDE
C      S=NOISE VARIANCE
C      AM=NOISE MEAN
C      IX=SEED
C      AA=FILTER COEFFICIENTS
C      W=WHITE GAUSSIAN NOISE GENERATED BY SUBROUTINE
C      D=DESIRED OUTPUT
C      Y=FILTER OUTPUT
C      E=ERROR(D - Y)
C      MXM = ARRAY SIZE
C      NXN = FILTER SIZE
C      Q=WEIGHTING FACTOR
C      P=INVERSE OF COVARIANCE MATRIX
C
C      ***** INPUT INITIAL VALUES *****
C      M=32
C      N=2
C      A=1.
C      AA(0)=0.
C      AA(1)=0.
C      AA(2)=0.
C      AA(3)=0.
C      Q=1.
C      S=1
C      AM=0
C      IX=65539
C
C      ***** BUILD INITIAL 'P' MATRIX *****
C      DO 1 K=0,(N**2-1)
C          DO 5 J=0,(N**2-1)
C              IF(K.EQ.J)THEN
C                  P(K,J)=100.
C              ELSE
C                  P(K,J)=0.0
C              ENDIF
C          CONTINUE
C      CONTINUE
C      OPEN (UNIT=63,FILE='RLSID2D2 DATA',STATUS='OLD')
C
C      ***** CALCULATE INPUT MATRIX (WHITE GAUSSIAN NOISE) *****
C      DO 10 K=-1,M-1
C          DO 20 J=-1,M-1
C              IF((K.LT.0).OR.(J.LT.0))THEN
C                  W(K,J)=0.

```

```

        ELSE
            CALL GAUSS(IX,S,AM,V)
            W(K,J)=V*A
        ENDIF
20    CONTINUE
10    CONTINUE
C
C    ***** CALCULATE ERROR BETWEEN ADAPTIVE FILTER OUTPUT
C    AND KNOWN FILTER OUTPUT.  USE THIS ERROR TO UPDATE
C    FILTER COEFFICIENTS AND THE 'P' MATRIX *****
DO 40 K=0,M-1
    DO 50 J=0,M-1
        L=K*M+J
        D=.4*W(K,J)+.6*W(K-1,J)-.3*W(K,J-1)+.2*W(K-1,J-1)
        Y=AA(0)*W(K,J)+AA(1)*W(K-1,J)+AA(2)*W(K,J-1)+
*        AA(3)*W(K-1,J-1)
        E=D-Y
        X(0)=W(K,J)
        X(1)=W(K-1,J)
        X(2)=W(K,J-1)
        X(3)=W(K-1,J-1)
        CALL RLS(P,X,AA,N,E,Q)
        PRINT 15, L,E,AA(0),AA(1),AA(2),AA(3)
        WRITE (63,15) L,E,AA(0),AA(1),AA(2),AA(3)
50    CONTINUE
40    CONTINUE
15    FORMAT (' ',2X,I3,2X,F9.5,2X,F9.5,2X,F9.5,2X,F9.5,2X,F9.5)
    STOP
    END
C
C
C    ***** TO COMPUTE THE GAIN MATRIX *****
SUBROUTINE RLS(P,X,AA,N,E,Q)
    DIMENSION P(0:3,0:3),X(0:3),AA(0:3),TEMY(0:3,0:3),
*    GAMA(0:3),TEMC(0:3)
    DO 30 L=0,3
        TEM=0.
        DO 20 K=0,3
20            TEM=TEM+X(K)*P(K,L)
30        GAMA(L)=TEM
        GAM=Q
        DO 40 K=0,3
40            GAM=GAM+GAMA(K)*X(K)
        DO 60 L=0,3
            TEM=0.
            DO 50 K=0,3
50                TEM=TEM+P(L,K)*X(K)
60            TEMC(L)=TEM
            DO 70 L=0,3
            DO 70 K=0,3
70            TEMY(L,K)=TEMC(L)*GAMA(K)
            DO 80 K=0,3
            DO 80 L=0,3
80            P(K,L)=(P(K,L)-(TEMY(K,L)/GAM))/Q
            DO 100 K=0,3
                TEM=0.

```

```
          DO 90 L=0,3
90      TEM=TEM+P(K,L)*X(L)
100     TEMC(K)=TEM
          DO 110 K=0,3
110     AA(K)=AA(K)+TEMC(K)*E
          Q=.99*Q+.01
          RETURN
          END
```


APPENDIX E. RLS ALGORITHM IMPLEMENTED IN A NOISE CANCELER

```

C      THIS IS A VAX/VMS FORTRAN PROGRAM THAT IMPLEMENTS A TWO
C      DIMENSIONAL 2X2 ADAPTIVE RLS FILTER WITHIN A NOISE
C      CANCELER
C
      INTEGER M,N,K,J
      BYTE B(0:127),BYTEE(0:127),BYTEU(0:127)
      INTEGER*4 INTE(0:127,0:127),INTU(0:127,0:127),IE,IU
      REAL*4 A,FMINE,FMAXE,FMINU,FMAXU
      REAL*4 AA(0:3),E(0:127,0:127),U(0:127,0:127)
      REAL*4 P(0:3,0:3),X(0:3),IM(0:127,0:127)
      REAL*4 W(-1:127,-1:127),Y(0:127,0:127)
C
C      *****VARIABLE DEFINITIONS*****
C      A=NOISE AMPLITUDE
C      S=NOISE VARIANCE
C      AM=NOISE MEAN
C      AA=FILTER COEFFICIENTS
C      IX=SEED
C      IM=INPUT IMAGE
C      W=WHITE GAUSSIAN NOISE GENERATED BY SUBROUTINE
C      U=SIGNAL PLUS NOISE(IM + W)
C      Y=FILTER OUTPUT
C      E=ERROR(U - Y)
C      Q=WEIGHTING FACTOR
C      P=INVERSE OF COVARIANCE MATRIX
C      FMINE,FMAXE,FMINU,FAMXU=PARAMETERS
C      TO BE USED TO CONVERT DECIMAL DATA TO BYTE DATA
C      MXM=ARRAY SIZE
C      NXN=FILTER SIZE
C
C      *****INITIAL VALUES*****
      M=128
      N=2
      Q=1.
      A=1.
      AA(0)=0.
      AA(1)=0.
      AA(2)=0.
      AA(3)=0.
      S=40.
      AM=0.
      IX=65539
      FMINE=1.E+10
      FMAXE=-1.E+10
      FMINU=1.E+10
      FMAXU=-1.E+10

```

```

C
C *****OPEN AN IMAGE FILE, CONVERT THE BYTE DATA INTO INTEGERS
C AND THEN PLACE THESE VALUES IN A MATRIX*****
OPEN (UNIT=1, NAME ='HOUS1G.DAT', TYPE ='OLD', ACCESS =
* 'DIRECT', RECORDSIZE=32, MAXREC=128)
DO 100 K=0,127
  READ(1,K+1) (B(L),L=0,127)
  DO 110 J=0,127
    IF(B(J).LT.0)THEN
      IM(K,J)=B(J)+256
    ELSE
      IM(K,J)=B(J)
    ENDIF
110    CONTINUE
100    CONTINUE
CLOSE (UNIT=1)

C
C ***** BUILD INITIAL 'P' MATRIX *****
DO 1 K=0,(N**2-1)
  DO 5 J=0,(N**2-1)
    IF(K.EQ.J)THEN
      P(K,J)=100.
    ELSE
      P(K,J)=0.0
    ENDIF
5    CONTINUE
1    CONTINUE

C
C *****ADD WHITE GAUSSIAN NOISE TO THE IMAGE AND SET THE
C VALUES OUTSIDE THE IMAGE TO ZERO*****
DO 10 K=-1,M-1
  DO 20 J=-1,M-1
    IF ((K.LT.0).OR.(J.LT.0))THEN
      W(K,J)=0.
      IM(K,J)=0.
    ELSE
      CALL GAUSS(IX,S,AM,V)
      W(K,J)=V*A
    ENDIF
    U(K,J)=IM(K,J)+W(K,J)
20    CONTINUE
10    CONTINUE

C
C ***** CALCULATE ERROR BETWEEN ADAPTIVE FILTER OUTPUT
C AND KNOWN FILTER OUTPUT. USE THIS ERROR TO UPDATE
C FILTER COEFFICIENTS AND THE 'P' MATRIX *****
DO 40 K=0,M-1
  DO 50 J=0,M-1
    L=K*M+J
    Y=AA(0)*W(K,J)+AA(1)*W(K-1,J)+AA(2)*W(K,J-1)+
* AA(3)*W(K-1,J-1)
    E(K,J)=U(K,J)-Y(K,J)
    EE=E(K,J)
    X(0)=W(K,J)
    X(1)=W(K-1,J)
    X(2)=W(K,J-1)

```

```

        X(3)=W(K-1,J-1)
        CALL RLS(P,X,AA,N,EE,Q)
50      CONTINUE
40      CONTINUE
C
C      *****CHANGE SIGNAL PLUS NOISE AND ERROR OUTPUT
C      INTO BYTE DATA AND WRITE TO A FILE*****
      OPEN (UNIT=2,NAME='ERROR.DAT',TYPE='NEW',ACCESS=
*      'DIRECT',RECORDSIZE=32,MAXREC=128)
      OPEN (UNIT=3,NAME='SIGNOISE.DAT',TYPE='NEW',ACCESS=
*      'DIRECT',RECORDSIZE=32,MAXREC=128)
      DO 500 I=0,127
        DO 550 J=0,127
          IF(E(I,J).LT.FMINE)THEN
            FMINE=E(I,J)
          ENDIF
          IF(E(I,J).GT.FMAXE)THEN
            FMAXE=E(I,J)
          ENDIF
550      CONTINUE
500      CONTINUE
      IF (FMINE.LT.0) THEN
        FMAXE=FMAXE-FMINE
      ENDIF
      DO 600 I=0,127
        DO 650 J=0,127
          IF(FMINE.LT.0)THEN
            E(I,J)=E(I,J)-FMINE
            E(I,J)=E(I,J)*255./FMAXE
          ELSE
            E(I,J)=(E(I,J)-FMINE)*255./FMAXE
          ENDIF
          IE=NINT(E(I,J))
          IF(IE.GT.127) THEN
            IE=IE-256
          ENDIF
          BYTEE(J)-IE
650      CONTINUE
        WRITE (2'I+1) (BYTEE(N),N=0,127)
600      CONTINUE
      DO 700 I=0,127
        DO 750 J=0,127
          IF(U(I,J).LT.FMINU)THEN
            FMINU=U(I,J)
          ENDIF
          IF(U(I,J).GT.FMAXU)THEN
            FMAXU=U(I,J)
          ENDIF
750      CONTINUE
700      CONTINUE
      IF (FMINU.LT.0) THEN
        FMAXU=FMAXU-FMINU
      ENDIF
      DO 400 I=0,127
        DO 450 J=0,127
          IF(FMINU.LT.0)THEN

```

```

        U(I,J)=U(I,J)-FMINU
        U(I,J)=U(I,J)*255./FMAXU
    ELSE
        U(I,J)=(U(I,J)=FMINU)*255./FMAXU
    ENDIF
    IU=NINT(U(I,J))
    IF(IU.GT. 127) THEN
        IU=IU-256
    ENDIF
    BYTEU(J)-IU
450    CONTINUE
        WRITE (3'I+1) (BYTEU(N),N=0,127)
400    CONTINUE
        CLOSE (UNIT=2)
        CLOSE (UNIT=3)
        STOP
        END
C
C ***** TO COMPUTE THE GAIN MATRIX *****
SUBROUTINE RLS(P,X,AA,N,E,Q)
    DIMENSION P(0:3,0:3),X(0:3),AA(0:3),TEMY(0:3,0:3),
*      GAMA(0:3),TEMC(0:3)
    DO 30 L=0,3
        TEM=0.
        DO 20 K=0,3
            TEM=TEM+X(K)*P(K,L)
20      GAMA(L)=TEM
        GAM=Q
        DO 40 K=0,3
            GAM=GAM+GAMA(K)*X(K)
40      DO 60 L=0,3
                TEM=0.
                DO 50 K=0,3
                    TEM=TEM+P(L,K)*X(K)
50      TEMC(L)=TEM
                DO 70 L=0,3
                    DO 70 K=0,3
80      TEMY(L,K)=TEMC(L)*GAMA(K)
                DO 80 K=0,3
                    DO 80 L=0,3
80      P(K,L)=(P(K,L)-(TEMY(K,L)/GAM))/Q
                DO 100 K=0,3
                    TEM=0.
                    DO 90 L=0,3
90      TEM=TEM+P(K,L)*X(L)
100     TEMC(K)=TEM
                DO 110 K=0,3
110     AA(K)=AA(K)+TEMC(K)*E
        Q=.99*Q+.01
        RETURN
    END

```


APPENDIX F. RLS ALGORITHM IMPLEMENTED IN AN ADAPTIVE LINE ENHANCER

C	THIS IS A VAX/VMS FORTRAN PROGRAM THAT IMPLEMENTS A TWO	APP00010
C	DIMENSIONAL 2X2 ADAPTIVE RLS FILTER WITHIN AN ADAPTIVE	APP00020
C	LINE ENHANCER	APP00030
C		APP00040
	INTEGER M,N,K,J	APP00050
	BYTE B(0:127),BYTEU(0:127),BYTEY(0:127)	APP00060
	INTEGER*4 INTY(0:127,0:127),INTU(0:127,0:127),IU,IY	APP00070
	REAL*4 A,FMINU,FMAXU,FMINY,FMAXY	APP00080
	REAL*4 AA(0:3),E(0:127,0:127),U(0:127,0:127)	APP00090
	REAL*4 P(0:3,0:3),X(0:3),IM(0:127,0:127)	APP00100
	REAL*4 W(-1:127,-1:127),Y(0:127,0:127)	APP00110
C		APP00120
C	*****VARIABLE DEFINITIONS*****	APP00130
C	A=NOISE AMPLITUDE	APP00140
C	S=NOISE VARIANCE	APP00150
C	AM=NOISE MEAN	APP00160
C	AA=FILTER COEFFICIENTS	APP00170
C	IX=SEED	APP00180
C	IM=INPUT IMAGE	APP00190
C	W=WHITE GAUSSIAN NOISE GENERATED BY SUBROUTINE	APP00200
C	U=SIGNAL PLUS NOISE(IM + W)	APP00210
C	Y=FILTER OUTPUT	APP00220
C	E=ERROR(U - Y)	APP00230
C	WW=DELAYED VERSION OF SIGNAL PLUS NOISE(U)	APP00240
C	Q=WEIGHTING FACTOR	APP00250
C	P=INVARSE OF COVARIANCE MATRIX	APP00260
C	FMINU,FAMXU,FMINY,FMAXY=PARAMETERS	APP00270
C	TO BE USED TO CONVERT DECIMAL DATA TO BYTE DATA	APP00280
C	MXM=ARRAY SIZE	APP00290
C	NXN=FILTER SIZE	APP00300
C		APP00310
C	*****INITIAL VALUES*****	APP00320
	M=128	APP00330
	N=2	APP00340
	Q=1.	APP00350
	A=1.	APP00360
	AA(0)=0.	APP00370
	AA(1)=0.	APP00380
	AA(2)=0.	APP00390
	AA(3)=0.	APP00400
	S=40.	APP00410
	AM=0.	APP00420
	IX=65539	APP00430
	FMINU=1.E+10	APP00440
	FMAXU=-1.E+10	APP00450
	FMINY=1.E+10	APP00460
	FMAXY=-1.E+10	APP00470
C		APP00480

C	*****OPEN AN IMAGE FILE, CONVERT THE BYTE DATA INTO INTEGERS	APP00490
C	AND THEN PLACE THESE VALUES IN A MATRIX*****	APP00500
	OPEN (UNIT=1, NAME ='HOUS1G.DAT', TYPE ='OLD', ACCESS =	APP00510
*	'DIRECT', RECORDSIZE=32, MAXREC=128)	APP00520
	DO 100 K=0,127	APP00530
	READ(1,K+1) (B(L),L=0,127)	APP00540
	DO 110 J=0,127	APP00550
	IF(B(J).LT.0)THEN	APP00560
	IM(K,J)=B(J)+256	APP00570
	ELSE	APP00580
	IM(K,J)=B(J)	APP00590
	ENDIF	APP00600
110	CONTINUE	APP00610
100	CONTINUE	APP00620
	CLOSE (UNIT=1)	APP00630
C		APP00640
C	***** BUILD INITIAL 'P' MATRIX *****	APP00650
	DO 1 K=0,(N**2-1)	APP00660
	DO 5 J=0,(N**2-1)	APP00670
	IF(K.EQ. J)THEN	APP00680
	P(K,J)=100.	APP00690
	ELSE	APP00700
	P(K,J)=0.0	APP00710
	ENDIF	APP00720
5	CONTINUE	APP00730
1	CONTINUE	APP00740
C		APP00750
C	*****ADD WHITE GAUSSIAN NOISE TO THE IMAGE AND SET THE	APP00760
C	VALUES OUTSIDE THE IMAGE TO ZERO*****	APP00770
	DO 10 K=-2,M-1	APP00780
	DO 20 J=-2,M-1	APP00790
	IF ((K.LT.0).OR.(J.LT.0))THEN	APP00800
	W(K,J)=0.	APP00810
	IM(K,J)=0.	APP00820
	ELSE	APP00830
	CALL GAUSS(IX,S,AM,V)	APP00840
	W(K,J)=V*A	APP00850
	ENDIF	APP00860
	U(K,J)=IM(K,J)+W(K,J)	APP00870
20	CONTINUE	APP00880
10	CONTINUE	APP00890
C		APP00900
C	*****COMPUTE THE DELAYED VERSION OF THE SIGNAL	APP00910
C	PLUS NOISE MATRIX(U)*****	APP00920
	DO 45 K=-1,127	APP00930
	DO 46 J=-1,127	APP00940
	IF((M.LT.-1).OR.(N.LT.-1) THEN	APP00950
	WW(K,J)=0.	APP00960
	ELSE	APP00970
	WW(K,J)=U(K-1,J)	APP00980
	ENDIF	APP00990
46	CONTINUE	APP01000
45	CONTINUE	APP01010
C		APP01020
C	***** CALCULATE ERROR BETWEEN ADAPTIVE FILTER OUTPUT	APP01030

C	AND KNOWN FILTER OUTPUT. USE THIS ERROR TO UPDATE	APP01040
C	FILTER COEFFICIENTS AND THE 'P' MATRIX *****	APP01050
	DO 40 K=0,M-1	APP01060
	DO 50 J=0,M-1	APP01070
	Y=AA(0)*WW(K,J)+AA(1)*WW(K-1,J)+AA(2)*WW(K,J-1)+	APP01080
*	AA(3)*WW(K-1,J-1)	APP01090
	E(K,J)=U(K,J)-Y(K,J)	APP01100
	EE=E(K,J)	APP01110
	X(0)=WW(K,J)	APP01120
	X(1)=WW(K-1,J)	APP01130
	X(2)=WW(K,J-1)	APP01140
	X(3)=WW(K-1,J-1)	APP01150
	CALL RLS(P,X,AA,N,EE,Q)	APP01160
50	CONTINUE	APP01170
40	CONTINUE	APP01180
C		APP01190
C	*****CHANGE SIGNAL PLUS NOISE AND FILTER OUTPUT	APP01200
C	INTO BYTE DATA AND WRITE TO A FILE*****	APP01210
	OPEN (UNIT=3,NAME='SIGNOISE.DAT',TYPE='NEW',ACCESS=	APP01220
*	'DIRECT',RECORDSIZE=32,MAXREC=128)	APP01230
	OPEN (UNIT=4,NAME='FILTERED.DAT',TYPE='NEW',ACCESS=	APP01240
*	'DIRECT',RECORDSIZE=32,MAXREC=128)	APP01250
	DO 700 I=0,127	APP01260
	DO 750 J=0,127	APP01270
	IF(U(I,J).LT.FMINU)THEN	APP01280
	FMINU=U(I,J)	APP01290
	ENDIF	APP01300
	IF(U(I,J).GT.FMAXU)THEN	APP01310
	FMAXU=U(I,J)	APP01320
	ENDIF	APP01330
750	CONTINUE	APP01340
700	CONTINUE	APP01350
	IF (FMINU.LT.0) THEN	APP01360
	FMAXU=FMAXU-FMINU	APP01370
	ENDIF	APP01380
	DO 400 I=0,127	APP01390
	DO 450 J=0,127	APP01400
	IF(FMINU.LT.0)THEN	APP01410
	U(I,J)=U(I,J)-FMINU	APP01420
	U(I,J)=U(I,J)*255./FMAXU	APP01430
	ELSE	APP01440
	U(I,J)=(U(I,J)-FMINU)*255./FMAXU	APP01450
	ENDIF	APP01460
	IU=NINT(U(I,J))	APP01470
	IF(IU.GT.127) THEN	APP01480
	IU=IU-256	APP01490
	ENDIF	APP01500
	BYTEU(J)-IU	APP01510
450	CONTINUE	APP01520
	WRITE (3'I+1) (BYTEU(N),N=0,127)	APP01530
400	CONTINUE	APP01540
	DO 800 I=0,127	APP01550
	DO 850 J=0,127	APP01560
	IF(Y(I,J).LT.FMINY)THEN	APP01570
	FMINY=Y(I,J)	APP01580
	ENDIF	APP01590

	IF(Y(I,J).GT.FMAXY)THEN	APP01600
	FMAXY=Y(I,J)	APP01610
	ENDIF	APP01620
850	CONTINUE	APP01630
800	CONTINUE	APP01640
	IF (FMINY.LT.0) THEN	APP01650
	FMAXY=FMAXY-FMINY	APP01660
	ENDIF	APP01670
	DO 900 I=0,127	APP01680
	DO 950 J=0,127	APP01690
	IF(FMINY.LT.0)THEN	APP01700
	Y(I,J)=Y(I,J)-FMINY	APP01710
	Y(I,J)=Y(I,J)*255./FMAXY	APP01720
	ELSE	APP01730
	Y(I,J)=(Y(I,J)=FMINY)*255./FMAXY	APP01740
	ENDIF	APP01750
	IY=NINT(Y(I,J))	APP01760
	IF(IY.GT.127) THEN	APP01770
	IY=IY-256	APP01780
	ENDIF	APP01790
	BYTEY(J)-IY	APP01800
950	CONTINUE	APP01810
	WRITE (4'I+1) (BYTEY(N),N=0,127)	APP01820
900	CONTINUE	APP01830
	CLOSE (UNIT=3)	APP01840
	CLOSE (UNIT=4)	APP01850
	STOP	APP01860
	END	APP01870
C		APP01880
C	***** TO COMPUTE THE GAIN MATRIX *****	APP01890
	SUBROUTINE RLS(P,X,AA,N,E,Q)	APP01900
	DIMENSION P(0:3,0:3),X(0:3),AA(0:3),TEMY(0:3,0:3),	APP01910
	* GAMA(0:3),TEMC(0:3)	APP01920
	DO 30 L=0,3	APP01930
	TEM=0.	APP01940
	DO 20 K=0,3	APP01950
20	TEM=TEM+X(K)*P(K,L)	APP01960
30	GAMA(L)=TEM	APP01970
	GAM=Q	APP01980
	DO 40 K=0,3	APP01990
40	GAM=GAM+GAMA(K)*X(K)	APP02000
	DO 60 L=0,3	APP02010
	TEM=0.	APP02020
	DO 50 K=0,3	APP02030
50	TEM=TEM+P(L,K)*X(K)	APP02040
60	TEMC(L)=TEM	APP02050
	DO 70 L=0,3	APP02060
	DO 70 K=0,3	APP02070
70	TEMY(L,K)=TEMC(L)*GAMA(K)	APP02080
	DO 80 K=0,3	APP02090
	DO 80 L=0,3	APP02100
80	P(K,L)=(P(K,L)-(TEMY(K,L)/GAM))/Q	APP02110
	DO 100 K=0,3	APP02120
	TEM=0.	APP02130

	DO 90 L=0,3	APP02140
90	TEM=TEM+P(K,L)*X(L)	APP02150
100	TEMC(K)=TEM	APP02160
	DO 110 K=0,3	APP02170
110	AA(K)=AA(K)+TEMC(K)*E	APP02180
	Q=.99*Q+.01	APP02190
	RETURN	APP02200
	END	APP02210

LIST OF REFERENCES

1. Andrews, H. and Hunt, B., *Digital Image Restoration*, Prentice-Hall, Inc., Reading, Massachusetts, 1987.
2. Gonzalez, R. and Wintz, P., *Digital Image Processing*, Addison-Wesley Publishing Co., Englewood Cliffs, New Jersey, 1977.
3. Rosenfeld, A. and Kak, A., *Digital Picture Processing*, Academic Press, New York, 1976.
4. Murphy, P. K., *Survey of Image Restoration Techniques*, The Johns Hopkins University, Applied Physics Laboratory, Laurel, Maryland, July 1988.
5. Treichler, J., Johnson, C., and Larimore, M., *Theory and Design of Adaptive Filters*, John Wiley and Sons Inc., New York, 1984.
6. Orfanidis, S. J., *Optimum Signal Processing, an Introduction*, Macmillan, 1985.
7. Giordano, A. and Hsu, F., *Least Square Estimation with Applications to Digital Signal Processing*, John Wiley and Sons Inc., New York, 1985.
8. Haykin, S., *Introduction To Adaptive Filters*, Macmillan Publishing Company, New York, 1984.
9. Hadhoud, M. M. and Thomas D. W., "The Two-Dimensional Adaptive LMS Algorithm," *IEEE Trans. Circuits Syst.*, Vol. CAS-35, no. 5, pp. 485-494, May 1988.
10. Ekstrom, M. P., "Realizable Wiener Filtering in Two Dimensions," *IEEE Trans. Acoust., Speech, Signal Processing*, Vol. ASSP-30, no. 30, pp. 31-39, February 1982.

11. Florian, S. and Feuer, A., "Performance Analysis of the LMS Algorithm with a Tapped Delay Line (Two Dimensional Case)," *IEEE Trans. Acoust., Speech, Signal Processing*, Vol. ASSP-34, no. 6, pp. 1542-1549, December 1986.
12. Bitmead, R. R. and Anderson B. D. O., "Performance of Adaptive Estimation Algorithms in Dependent Random Environments," *IEEE Trans. Automatic Control*, Vol. AC-25, pp. 788-794, August 1980.
13. Widrow, B. and Stearns, S. D., *Adaptive Signal Processing*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1985.
14. Wellstead, P. E. and Caldas Pintos J. R., "Self-tuning Filters and Predictors for Two Dimensional Systems," *International Journal of Control*, Vol. 42, no. 2, pp. 457-478, 1985.

INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Technical Information Center Cameron Station Alexandria, VA 22304-6145	2
2. Library, Code 0142 Naval Postgraduate School Monterey, CA 93943-5002	2
3. Chairman Electrical and Computer Engineering Department, Code 62 Naval Postgraduate School Monterey, CA 93940-5000	1
4. Dr. Murali Tummala Electrical and Computer Engineering Department, Code 62 Naval Postgraduate School Monterey, CA 93940-5000	2
5. Dr. Charles Therrien Electrical and Computer Engineering Department, Code 62 Naval Postgraduate School Monterey, CA 93940-5000	1
6. Professor Will Gersch Department of Information and Computer Sciences University of Hawaii Honolulu, HI 96822	1
7. Patrol Squadron Forty-Seven Attn: Lcdr. A. R. Topp FPO San Francisco San Francisco, CA 96601-5920	1
8. Naval Underwater Systems Center Attn: Lt. R. E. Reinke New London, CT 06320	1
9. Patrol Squadron Nine Attn: Lcdr. Steven L. Wilstrup FPO San Francisco San Francisco, CA 96601-5905	1
10. Ocean Systems Center Commander Naval Ocean Systems Center San Diego, CA 92152-5000	1

Thesis
W644329 Wilstrup
c.1 Adaptive algorithms
for two dimensional
filtering.

Thesis
W644329 Wilstrup
c.1 Adaptive algorithms
for two dimensional
filtering.



thesW644329

Adaptive algorithms for two dimensional



3 2768 000 81691 2
DUDLEY KNOX LIBRARY